17 Status and Perspectives of the Standard Model

As described in the preceding chapters, the $SU(3)_c \times SU(2)_L \times U(1)_Y$ local gauge theory of strong and electroweak interactions accommodates all known elementary particles and incorporates the proven symmetries and successes of quantum electrodynamics, the Fermi–Feynman–Gell-Mann $V - A$ theory of weak interaction and the quark model. Despite intense experimental scrutiny, the theory has not yet displayed any signs of discrepancies or inconsistencies. The list of the predictions that are successfully tested is impressive; particularly noteworthy are the demonstrations of the existence and the properties of the neutral currents, of the $W^{\pm}$ and Z^0 gauge bosons, and of the τ lepton, the charm, bottom and top quarks.

The recent discovery of the top quark, in particular, is quite remarkable in this respect. Although the top quark is anticipated to exist as the weak-isospin partner of the bottom quark, the Standard Model gives no clear indications as to its mass. However, various ingenious theoretical arguments have succeeded in placing more and more stringent constraints on this property. Thus, at the beginning, a lower bound of $40\,\mathrm{GeV} < m_t$ was inferred from the observation of the B^0–$\overline{B}^0$ mixing. Later, sophisticated calculations at the quantum level of the basic properties of the gauge bosons gave results that could be compared with data at the 10^{-3} level of accuracy, which led to improved predictions of the top quark mass. In particular, extremely precise data on the mass and width of Z^0, and on the forward–backward asymmetry of fermions produced in the decay $Z^0 \to f + \overline{f}$ were available which in turn determine $\sin^2\theta_W$ precisely.

Fig. 17.1. Electroweak corrections to the W and Z gauge boson propagators by the top quark

The most important contributions to these processes beyond the tree level come from the Z^0 and W self-energies (Fig. 17.1) in which the top

quark plays an essential role in making more precise the value of $\sin^2\theta_{\mathrm{W}}$ through its contribution:

$$\delta_{\sin^2\theta_{\mathrm{W}}} = \frac{-3\alpha_{\mathrm{em}}}{16\pi\sin^2\theta_{\mathrm{W}}}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2}\,. \tag{17.1}$$

From this result, one infers[1] the top quark mass to be $m_{\mathrm{t}} = 169 \pm 24$ GeV, to be compared with its experimental value of 180 ± 12 GeV.

Given its successes, the Standard Model should be accepted as a proven paradigm against which future experimental findings and alternative theories must be confronted. Yet, in spite of its many achievements, the model leaves unresolved several fundamental issues, most of which revolve around the breaking of the electroweak gauge symmetry. As we know, such a symmetry breaking is a necessary condition for the bosons and fermions in the theory to acquire masses by absorption of the Goldstone bosons associated with a spontaneous breaking of symmetry. But we do not have any idea what breaks it. In the Standard Model, this spontaneous symmetry breaking is realized via an ingenious although artificial process called the Higgs mechanism. The mechanism leaves behind a scalar boson as its distinctive signature, a signature as yet undetected. Therefore, the experimental observation of the Higgs boson is crucial for the confirmation of the Standard Model. We will examine in the following section some salient properties of the standard Higgs boson through its productions and decays. We next indicate some possible extensions of the standard electroweak model.

17.1 Production and Decay of the Higgs Boson

As explained in Chap. 9, a doublet of complex scalar fields $\boldsymbol{\phi}(x)$ is postulated in the Standard Model. The dynamics of this scalar field is governed by the potential $V(\boldsymbol{\phi}) = \lambda(\boldsymbol{\phi}^\dagger\boldsymbol{\phi})^2 + \mu^2\boldsymbol{\phi}^\dagger\boldsymbol{\phi}$, on which the condition $\mu^2 < 0$ is imposed in order to spontaneously break the gauge symmetry and give masses to the gauge bosons, the quarks, and the charged leptons. These masses can be expressed in terms of the vacuum expectation value $\sqrt{2} < \boldsymbol{\phi} >= v = \sqrt{-\mu^2/\lambda}$ and nine arbitrary Yukawa couplings $C_{\mathrm{f}} = \sqrt{2}m_{\mathrm{f}}/v$, where f stands for one of the six quarks or one of the three charged leptons. Since their right-handed components are absent, the neutrinos are decoupled from the Higgs field and remain massless. Note that $v = (\sqrt{2}\,G_{\mathrm{F}})^{-1/2} \approx 246$ GeV.

The complex scalar field doublet $\boldsymbol{\phi}$ has by definition four real components which may be written as

$$\boldsymbol{\phi} = \begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi^1 + \mathrm{i}\varphi^2\\ \varphi^3 + \mathrm{i}\varphi^4\end{pmatrix}\,. \tag{17.2}$$

[1] Hollick, W. and Marciano, W. in *Precision Tests of the Standard Electroweak Model* (ed. Langacker, P.). World Scientific, Singapore 1995

The three massless Goldstone bosons $w^{\pm} = (\varphi^1 \mp \mathrm{i}\varphi^2)/\sqrt{2}$ and $z = \varphi^4$ interact with the initially massless gauge fields A^1_μ, A^2_μ, A^3_μ, and B_μ to become the longitudinal components of the massive physical gauge bosons $\mathrm{W}^{\pm}$ and Z^0, while the component $\varphi^3 - v \equiv \mathrm{H}$ emerges as the physical neutral Higgs scalar boson with mass $M_{\mathrm{H}} = |\mu|\sqrt{2} = v\sqrt{2\lambda}$. This mass is a free parameter in the model. In a sense, the discovery of $\mathrm{W}^{\pm}$ and Z^0 is equivalent to that of $w^{\pm}$ and z. The detection of the Higgs scalar boson stands out as a major goal of high-energy physics today and in the near future.

How could H be discovered ? Since the production of H and its subsequent decay depend on the Higgs boson mass, we first discuss the constraints on this parameter. The large electron–positron collider (LEP) experiments at CERN give 71 GeV as a lower bound for the Higgs boson mass. This lower bound is derived from the negative result to find H in the reaction $\mathrm{e}^+ + \mathrm{e}^- \to \mathrm{Z}^0 + \mathrm{H}$, which is shown by the diagram in Fig. 17.2. The Higgs boson could also be produced in proton–proton and proton–antiproton high-energy collisions; here the basic processes are the quark-pair annihilation $\mathrm{q}\overline{\mathrm{q}} \to V^* \to \mathrm{V} + \mathrm{H}$ and the gluon fusion $\mathrm{gg} \to \mathrm{H}$ (Fig. 17.3), with the latter being the dominant production mechanism.

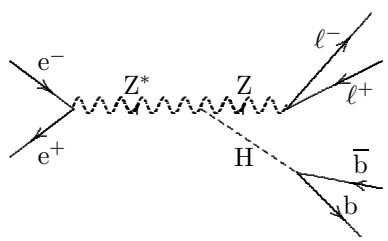

Fig. 17.2. Production of H by $\mathrm{Z}^* \to \mathrm{Z} + \mathrm{H}$ and subsequent decay of Z and H

In general, the presence of a Higgs boson could be uncovered through its decay products, which may proceed through one of the following modes, with partial widths predicted by the simplest calculations in the Standard Model:

(i) $\mathrm{H} \to \mathrm{V} + \mathrm{V}$, $\mathrm{H} \to \mathrm{V} + \mathrm{V}^*$ (V is a real vector boson W or Z, and V^* a virtual vector boson):

$$\Gamma(\mathrm{H} \to \mathrm{Z}^0 + \mathrm{Z}^0) = \frac{1}{2}\,\frac{\alpha_{\mathrm{em}}}{16\sin^2\theta_{\mathrm{W}}}\,\frac{M_{\mathrm{H}}^3}{M_{\mathrm{W}}^2}\sqrt{1-x_{\mathrm{z}}}\,[1 - x_{\mathrm{z}} + \tfrac{3}{4}x_{\mathrm{z}}^2]\,;$$

$$\Gamma(\mathrm{H} \to \mathrm{W}^+ + \mathrm{W}^-) = \frac{\alpha_{\mathrm{em}}}{16\sin^2\theta_{\mathrm{W}}}\,\frac{M_{\mathrm{H}}^3}{M_{\mathrm{W}}^2}\sqrt{1-x_{\mathrm{w}}}\,[1 - x_{\mathrm{w}} + \tfrac{3}{4}x_{\mathrm{w}}^2]\,; \qquad (17.3)$$

with $x_{\mathrm{v}} = 4M_{\mathrm{V}}^2/M_{\mathrm{H}}^2$, the factor $\frac{1}{2}$ comes from identical $\mathrm{Z}^0 + \mathrm{Z}^0$ bosons.

(ii) $\mathrm{H} \to \mathrm{f} + \overline{\mathrm{f}}$ (f is a lepton or a quark):

$$\Gamma(\mathrm{H} \to \mathrm{f} + \overline{\mathrm{f}}) = \frac{\alpha_{\mathrm{em}}\,N_{\mathrm{c}}}{8\sin^2\theta_{\mathrm{W}}}\,\frac{m_{\mathrm{f}}^2}{M_{\mathrm{W}}^2}\left(1 - \frac{4m_{\mathrm{f}}^2}{M_{\mathrm{H}}^2}\right)^{3/2} M_{\mathrm{H}}\,, \qquad (17.4)$$

where $N_{\rm c} = 3$ for quarks and 1 for leptons.
(iii) $\mathrm{H} \to \mathrm{g} + \mathrm{g}$ (g is a gluon):

$$\Gamma(\mathrm{H} \to \mathrm{g}+\mathrm{g}) = \frac{\alpha_{\rm em}}{\sin^2\theta_{\rm W}} \frac{\alpha_s^2}{32\pi^2} \frac{M_{\rm H}^3}{M_{\rm W}^2} \left| \sum_{\rm q} \tau_{\rm q}[1 + (1-\tau_{\rm q})\, h(\tau_{\rm q})] \right|^2 ,$$

$$h(\tau) = \left[\arcsin(\sqrt{1/\tau})\right]^2 \quad \text{or} \quad h(\tau) = -\tfrac{1}{4}\left[\log\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - \mathrm{i}\pi\right]^2 , \quad (17.5)$$

depending on $\tau \geq 1$ or < 1, where $\tau_{\rm q} = 4\, m_{\rm q}^2/M_{\rm H}^2$, and q stands for quarks.
(iv) $\mathrm{H} \to \gamma + \gamma$:

$$\Gamma(\mathrm{H} \to \gamma+\gamma) = \frac{\alpha_{\rm em}^3}{256\pi^2 \sin^2\theta_{\rm W}} \frac{M_{\rm H}^3}{M_{\rm W}^2} \left| \sum_j N_{c,j}(e_j^2)\, F_j(\tau_j) \right|^2 ,$$

$$F_1(\tau_1) = 2 + 3\tau_1[1 + (2-\tau_1)\, h(\tau_1)] , \qquad F_0(\tau_0) = \tau_0\,[1 - \tau_0\, h(\tau_0)] ,$$
$$F_{1/2}(\tau_{1/2}) = -2\tau_{1/2}\,[1 + (1-\tau_{1/2})\, h(\tau_{1/2})] . \quad (17.6)$$

Contributions to the amplitude for $\mathrm{H} \to \gamma\gamma$ in the Standard Model are represented by five loop diagrams, one with fermions, two with the gauge bosons W, and two with the scalar Goldstone bosons w (those which become the longitudinal components of W) circulating in loops. In (6) the sum over j stands for the contributions of the spin 0, $\frac{1}{2}$, and 1 of these internal virtual particles. The electric charge e_j is in units of e, and $N_{c,j}$ is the color multiplicity, i.e. 1 for W, w and leptons, and 3 for quarks. The function $h(\tau)$ is given in (5), and $\tau_j = 4m_j^2/M_{\rm H}^2$.

Since the decay amplitudes (5) and (6) only come at $\mathcal{O}(gg_s^2)$ and $\mathcal{O}(g^3)$ perturbative orders, they are also sensitive to the physics beyond the Standard Model due to possibly unknown particles circulating in loop diagrams.

Note that compared with $\mathrm{H} \to \mathrm{VV}$ or $\mathrm{H} \to \mathrm{VV}^*$, the rates for $\mathrm{H} \to \mathrm{f\bar{f}}$, $\mathrm{H} \to \mathrm{gg}$, and $\mathrm{H} \to \gamma\gamma$ are reduced respectively by $\sim m_{\rm f}^2/M_{\rm H}^2$, $\alpha_s^2/2\pi^2$, and $\alpha_{\rm em}/16\pi^2$. Which of those modes would actually occur depends on the energy available. Thus, it is convenient from the observational point of view to divide the possible mass range of H into three distinct regions.

(a) In the mass range $M_{\rm V} < M_{\rm H} < 2M_{\rm V}$ (where V stands for W or Z), all modes, *except* the production of real vector bosons and top quarks, could occur, but the dominant decay mode, $\mathrm{H} \to \mathrm{V} + \mathrm{V}^*$, where V^* is an off-shell virtual V, is similar to the production mechanism $\mathrm{Z}^* \to \mathrm{Z} + \mathrm{H}$ of Fig. 17.2. The LEP and the $\mathrm{p\bar{p}}$ collider at Fermilab can cover the range $90 < M_{\rm H} < 130$ GeV. In this mass range, the productions and decays are dominated by the mechanism $\mathrm{V}^* \to \mathrm{V} + \mathrm{H}$.

For the relatively low Higgs mass in this range, the modes $\mathrm{H} \to \mathrm{Z} + \gamma$ and $\mathrm{H} \to \gamma + \gamma$ are interesting because of their distinctive signatures. The partial width of $\mathrm{H} \to \gamma + \gamma$ is reduced by a factor proportional to $\alpha_{\rm em}/16\pi^2$ to less than 2% of $M_{\rm H}$, even for $M_{\rm H}$ as large as $2M_{\rm Z}$.

(b) If $2M_{\rm V} < M_{\rm H} < 2m_{\rm t}$, the Higgs boson may decay into *real* $\rm W^+ + W^-$ or $\rm Z^0 + Z^0$, mostly into their longitudinal components ($w^\pm$ or z) since these states arise from the Higgs mechanism. An explicit calculation gives

$$\frac{\Gamma(\rm H \to V_T + V_T)}{\Gamma(\rm H \to V_L + V_L)} = \frac{x_{\rm v}^2}{2(1-\frac{1}{2}x_{\rm v})^2} \qquad \text{with} \quad x_{\rm v} = \frac{4M_{\rm V}^2}{M_{\rm H}^2}\,,$$

where $\rm V_T$ ($\rm V_L$) is a transversely (longitudinally) polarized gauge boson V. The third power dependence of the width on $M_{\rm H}$ in (3) makes the width of H very large for a heavy Higgs boson. Numerically, the sum of the decay widths of H into $\rm Z^0\ Z^0$ and $\rm W^+\ W^-$ is $\approx \frac{1}{2}(M_{\rm H}/{\rm TeV})^3\times$ TeV. However for $M_{\rm H} \approx 300$ GeV, it is important to note that the Higgs boson width is still narrow; with $M_{\rm H} \approx 300$ GeV, the width of H decay into the two dominant channels $\rm W^+W^-$ and $\rm Z^0\ Z^0$ is less than 10 GeV. These modes, having the signatures of four highly energetic charged leptons in the final state (by the subsequent decays of VV), would be spectacular. The next decay mode is the $\rm b\overline{b}$-quark pair.

(c) A very heavy boson, $M_{\rm H} > 2m_{\rm t}$, could be seen at the future proton–proton large hadron collider (LHC) at CERN. The fusion of two gluons as depicted by the diagram in Fig. 17.3 is likely to be the dominant production mechanism with an effective Higgs boson–gluon–gluon coupling. This coupling may be obtained by computing the triangle loop diagram (Fig. 17.3) in which the top quark contribution dominates (Problem 17.3).

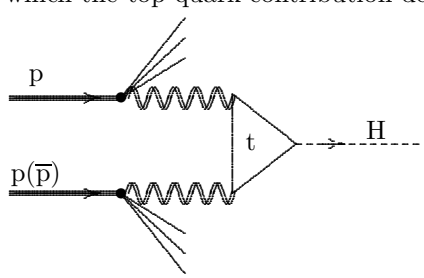

Fig. 17.3. Production of the Higgs boson by fusion of two gluons

17.2 Why go Beyond the Standard Model?

If the Standard Model works so well, why must there be any new physics at all? Actually, there are many reasons why the electroweak theory cannot have the final word. For one, the Standard Model is largely silent on the issue of the origin of the Higgs boson, and this is because the spontaneous breaking of the electroweak $\rm SU(2)\times U(1)$ symmetry is postulated in the model by the device of introducing a potential of scalar fields, $V(\phi)$, constructed just so that it can lead to such a breaking. And the condition for this to happen is $\mu^2 < 0$. This raises the question, what drives μ^2 negative? Clearly the

answer, if any, lies beyond the realm of the model, perhaps in some more complex, even fundamental mechanism. The spontaneous breaking of the electroweak $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry may possibly be realized by an unknown mechanism where the scalar fields would not be needed.

Let us recall that the Higgs mechanism of the Standard Model, though inspired by Anderson's approach[2] of the screening current in solid state physics, lacks a dynamical basis, in contrast to superconductivity. As pointed out by Anderson, there are several phenomena in solid state physics which could be interpreted in terms of an effective, massive electromagnetic field, a typical example being the Meissner effect. If a normal metal is cooled down below its superconducting transition temperature, the flux of an external applied magnetic field will be abruptly expelled. The expulsion of the magnetic field from a conductor can be interpreted as if the photon were massive, with a mass M, such that the magnetic field $\boldsymbol{B}(x)$ derived from the solution of equation (8) behaves like $\boldsymbol{B}(x) = \boldsymbol{B}_0\,\mathrm{e}^{-Mx}$, and the field can only penetrate the conductor within a distance x of order M^{-1}. The range M^{-1} is called the screening length in condensed matter physics.

For a photon described by an electromagnetic field $A^\rho(t,\boldsymbol{x})$ or a static field $\boldsymbol{A}(\boldsymbol{x})$ that obeys the Maxwell equation

$$\left[\frac{\partial^2}{\partial t^2} - \nabla^2\right] A^\rho = J^\rho \quad\Longrightarrow\quad \nabla^2\,\boldsymbol{A} = -\boldsymbol{J} \tag{17.7}$$

to acquire a mass M, it must place itself in a situation where it satisfies the equation

$$\left[\frac{\partial^2}{\partial t^2} - \nabla^2 + M^2\right] A^\rho = 0 \quad\Longrightarrow\quad \nabla^2\,\boldsymbol{A} = M^2\,\boldsymbol{A}\ . \tag{17.8}$$

In other words, the driving effective current $\boldsymbol{J}$ must be proportional to the field $\boldsymbol{A}$, i.e. $\boldsymbol{J} = -M^2\,\boldsymbol{A}$ (or its covariant generalization J^ρ is proportional to A^ρ). This relation which connects the current to the gauge field constitutes the Anderson mechanism that generates an effective mass for the photon, from which the Higgs model finds inspiration.

A specific example of magnetic screening is provided by a single non-relativistic particle of mass m and charge e. The associated current $\boldsymbol{J}$ is given by the usual quantum mechanics rules

$$\boldsymbol{J}_{\mathrm{free}} = \frac{e}{2\,m}\left[\psi^*(-\mathrm{i}\nabla\psi) + \mathrm{h.c.}\right]\ . \tag{17.9}$$

In the presence of an external electromagnetic field, ∇ is replaced by the covariant derivative $\mathbf{D} = \nabla - \mathrm{i}e\boldsymbol{A}$, so that $\boldsymbol{J}$ becomes

$$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{free}} - \frac{e^2}{m}|\psi|^2\boldsymbol{A}\ . \tag{17.10}$$

[2] Anderson, P. W., Phys. Rev. **130** (1963) 439

The term proportional to $\boldsymbol{A}$ will therefore yield an effective mass $M^2 = e^2|\psi|^2/m$ for the photon, and induces a diamagnetic screening of atomic electrons, characterized by a screening length $\sqrt{m}/|e|\,|\psi|$.

If this idea is applied to a superconductor, a physical interpretation must be found for the wave function ψ. Cooper showed that, in spite of the repulsive Coulomb force between them, two electrons with opposite spins can bind under certain circumstances, no matter how weak their mutual attraction is (the attraction force comes from the vibrations of lattice ions in the superconductor). Such a stable pair is called a Cooper pair which behaves as a boson of charge $-2e$. And the wave function ψ that causes screening represents just the macroscopic coherent state of the plasma of Cooper electron pairs which undergo the Bose condensation.

Applied to particle physics, the vacuum expectation value $\langle\phi\rangle = v/\sqrt{2}$ plays the role of the condensated Cooper pairs, $\langle\overline{\psi}\psi\rangle$. The current of the scalar Higgs field $J^\rho = \boldsymbol{\phi}^*(\partial^\rho\boldsymbol{\phi})+$ h.c. combined with the covariant derivative $D^\rho = \partial^\rho + \mathrm{i}\,g\mathrm{W}^\rho$ contains a term $(\frac{1}{2}g\,v)^2\mathrm{W}^\rho$. This term of J^ρ being proportional to the gauge field W^ρ, is responsible for its mass. However, contrary to superconductivity where the electron pairing can be derived from the principles of quantum mechanics, the Higgs mechanism of particle physics is described by an ad hoc assumption, $\mu^2 < 0$, that yields $v \neq 0$. The origin and the nature of this spontaneous symmetry-breaking must be found. Following the discovery of the top quark, it has been speculated that these very heavy quarks would somehow condensate and the resulting pairs would be the Cooper pairs of the electroweak particle physics. If so, the force that binds the top quark would be novel and should be found outside the Standard Model. The Standard Model would be the effective form of a certain more fundamental theory, valid only below a certain critical energy which defines its domain of validity. The new physics lies beyond this domain.

17.3 The Standard Model as an Effective Theory

It would be instructive to have another look at the 'old' weak interaction as formulated by Fermi, Feynman and Gell-Mann, Sudarshan and Marshak. As is well known, this theory is an effective low-energy limit of the Glashow–Salam–Weinberg electroweak theory. Starting from a four-fermion interaction

$$\frac{G_\mathrm{F}}{\sqrt{2}}[\overline{\psi}\gamma^\mu(1-\gamma_5)\psi]\,[\overline{\psi}\gamma_\mu(1-\gamma_5)\psi]\ , \tag{17.11}$$

one may calculate, for example, the fermion–fermion scattering amplitude f+f $\to$ f+f in the lowest order of G_F. Since the amplitude is dimensionless, a simple dimensional argument shows that the leading term must be proportional to $G_\mathrm{F}E^2$, up to a numerical factor, where E is the center-of-mass energy. At high energy, where the fermion masses may be neglected, explicit calculation gives $G_\mathrm{F}E^2(1+\cos\theta)$, i.e. at most only s- and p-partial waves contribute. There is a quick way to estimate the critical energy E_cr at which

the Fermi theory becomes inconsistent and useless. It is a consequence of *unitarity* or probability conservation that any partial wave of the two-body scattering amplitude (parameterized by $e^{i\delta_l}\sin\delta_l$) must be bounded by 1, the real part of the partial wave amplitude is bounded by $\frac{1}{2}$. This yields

$$E_{\rm cr} \approx \left(\frac{\sqrt{2}\,\pi}{G_{\rm F}}\right)^{1/2} \approx 600\ {\rm GeV}\ . \tag{17.12}$$

This value tells us that the Fermi theory as described by (11) is at most a phenomenological model valid only at energies below $E_{\rm cr}$. Above this limit, the theory violates unitarity and becomes self-contradictory.

For the Fermi theory, the new physics is represented by the appearance of the W and Z bosons (Fig. 17.4), which must exist to regulate the growth of the four-fermion amplitude at high energy.

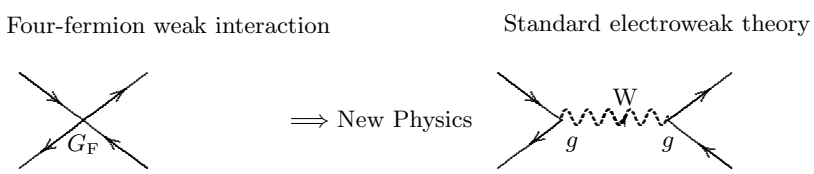

Fig. 17.4. For the Fermi theory, "new physics" is represented by the W boson

The presence of these intermediate bosons radically changes the energy dependence of the amplitude $G_{\rm F}E^2$ of the Fermi theory, so that

$$G_{\rm F}\,E^2 \Longrightarrow \frac{g^2E^2}{E^2-M_{\rm W}^2} \tag{17.13}$$

and tends to a constant g^2 as $E\to\infty$. The unitarity constraint is well illustrated by the neutrino–electron cross-section given by (12.42)–(12.44). Without the W-boson propagator, this cross-section would increase linearly with energy and would violate Froissart–Martin bound.

The expansion parameter is a small coupling g with $g^2/4\pi<1$, so that the theory is predictive at all energies. The fact that new physics ($\sim$ 100 GeV) enters long before the critical energy 600 GeV can be related to a small coupling strength g.

17.3.1 Problems with the Standard Model

For the Fermi theory, the new physics which resolves the disastrous growth of the partial wave amplitudes is contained in the Standard Model. However, the electroweak model is not without its own problems.

To see this, we now consider the scattering W+W$\to$ W+W of the gauge bosons as shown in Fig. 17.5. At high energy, the longitudinal component $\varepsilon_{\rm L}^\mu$ of the W dominates the amplitude, as can be seen directly from $\varepsilon_{\rm L}^\mu\sim p^\mu/M_{\rm W}$.

With only the gauge bosons Z^0 and γ exchanged in the s- and t-channels (Fig. 17.5a), the W–W scattering amplitude $\mathcal{M}$ grows like g^2E^2/M_W^2, similar to $G_F E^2$ of the f+f $\to$ f+f scattering amplitude. The reason for $\mathcal{M} \sim E^2$ is that the three-vertex of the gauge bosons depends on momenta. Including the Higgs boson exchange contribution (Fig. 17.5b) eliminates the rising $\sim E^2$-behavior of $\mathcal{M}$, and the resulting amplitude W+W$\to$ W+W becomes proportional to the Higgs boson mass, i.e. behaves like $g^2M_H^2/M_W^2$ (Problem 17.2). The similarity with (13) is striking:

$$\frac{g^2}{32\pi}\frac{E^2}{M_W^2} \Longrightarrow \frac{g^2}{32\pi}\frac{M_H^2}{M_W^2} = \frac{\lambda}{4\pi}\ , \tag{17.14}$$

which in turn implies that the amplitude is proportional to the self-coupling constant λ of the Higgs field. As long as λ is small enough, i.e. as long as the Higgs boson mass $M_H < 700$ GeV (see below), everything may be acceptable from the point of view of the perturbation theory, except that we still do not understand why the parameter μ^2 is negative. For $\lambda \gg 1$ or $M_H \geq 700$ GeV, the Standard Model becomes unpredictive and we do not even know whether nature provides a Higgs boson to regulate the growth in energy of the W–W scattering amplitude.

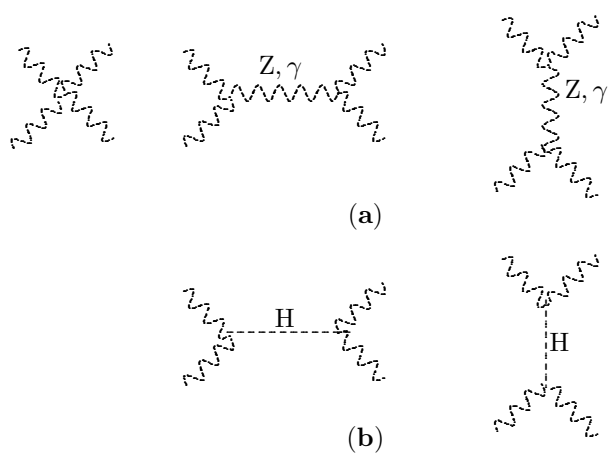

Fig. 17.5. W+W $\to$ W+W scattering (**a**) without, and (**b**) with the Higgs boson exchanged

Let us summarize the important points that can be drawn from this brief analysis of the W–W scattering.

(i) The Higgs boson is necessary to regulate the E^2-growth of the amplitude, otherwise unitarity would be violated.

(ii) If a Higgs boson can be found in future experiments with a relatively small mass, say $M_H < v = 246$ GeV, then we are in the weakly coupled

regime of λ and the theory remains consistent. But the question why $\mu^2 < 0$ will remain unanswered.

(iii) However, if the Higgs boson is too heavy, i.e. if the coupling constant $\lambda \gg 1$, terms of higher orders in λ become increasingly more important and will get out of control, and the perturbative approach loses its usefulness.

All of these considerations strongly hint at the possibility that the electroweak Standard Model would somehow be embedded in a more fundamental theory. Which one, that is the most compelling question of today's particle physics. The upper bound of ≈ 700 GeV considered as the critical Higgs boson mass which separates the weakly coupled regime from the strongly coupled regime can be estimated from the renormalization group equation.

17.3.2 Renormalization Group Equation Analysis

In (2), the complex scalar doublet, conveniently considered as a set of four real fields, is governed by the self-interaction $\lambda(\boldsymbol{\phi}^\dagger\boldsymbol{\phi})^2 = \frac{\lambda}{4}(\sum_i \varphi_i^2)^2, i = 1, \ldots, 4$. In the $\lambda\phi^4/4!$ theory for one real scalar field, the Callan–Symanzik β function is $3\lambda^2/16\pi^2$ (Problem 15.4). In $\lambda(\boldsymbol{\phi}^\dagger\boldsymbol{\phi})^2$ considered here, an additional combinatorial factor $8 = 4 \times 2$ enters and the corresponding $\beta(\lambda)$ function for the Higgs boson field is calculated to be $\beta(\lambda) = 8 \times (3\lambda^2/16\pi^2) = 3\lambda^2/2\pi^2$. The renormalization group equation for λ is

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\log(Q/v)} = \frac{3}{2\pi^2}\lambda^2 \,. \tag{17.15}$$

In this β-function, we have neglected contributions from fermions and gauge bosons coupled to the Higgs boson, since we are interested only in the limit of large λ (large M_H) for which the λ^2 term in the β-function dominates. The solution to (15) can be rewritten as

$$\frac{1}{\lambda(v)} - \frac{1}{\lambda(Q)} = \frac{3}{2\pi^2}\log(Q/v) \tag{17.16}$$

$$\text{or}\;\; \lambda(Q) = \frac{\lambda}{1 - \dfrac{3\lambda}{2\pi^2}\log(Q/v)} \;\;,\;\; \lambda(v) \equiv \lambda \,. \tag{17.17}$$

This result shows that, regardless of how small λ is, the coupling strength $\lambda(Q)$ grows with increasing energy. Using $M_\mathrm{H}^2 = 2\lambda\, v^2$, $\lambda(Q)$ becomes infinite at the scale $Q = E_\mathrm{cr}$ where the denominator of (17) vanishes, i.e.

$$\frac{E_\mathrm{cr}}{v} = \exp\left(\frac{4\pi^2 v^2}{3M_\mathrm{H}^2}\right) . \tag{17.18}$$

The formula (18) is remarkable because M_H is in the denominator of an exponential which makes the correlation between E_cr and M_H particularly interesting. Table 17.1 shows M_H for some selected values of E_cr. For small

$M_{\rm H} < 150$ GeV, the critical energy scale $E_{\rm cr}$ is very high $\sim 10^{18}$ GeV, and the Higgs model is valid at this high-energy scale. However, for large $M_{\rm H} \approx 700$ GeV, the critical energy $E_{\rm cr}$ decreases exponentially so quickly that it nearly reaches $M_{\rm H}$. In this case, the running coupling constant λ blows up for $M_{\rm H}$ not far from 700 GeV. Larger values of the Higgs boson mass are self-contradictory, since the cutoff $E_{\rm cr}$ by definition cannot be smaller than the effective upper limit of the mass spectrum of the theory.

Table 17.1. $E_{\rm cr}$ versus $M_{\rm H}$

$M_{\rm H}$ in GeV	$E_{\rm cr}$ in GeV
150	6×10^{17}
200	1×10^{11}
300	2×10^{6}
500	6×10^{3}
700	1×10^{3}

The illustrative result in (18) may be interpreted as follows. Either the Higgs model is an effective Lagrangian of some unknown strong interaction at the scale $E_{\rm cr}$, or at energies below $E_{\rm cr}$, the Standard Model is embedded in a more fundamental theory where $E_{\rm cr}$ acts as a cutoff.

Whatever the mechanism of the electroweak symmetry breaking, it would have very little impact on the precision electroweak data. Veltman has shown that the Higgs boson contribution to radiative corrections is screened by a slowly-varying logarithm function. For instance, the radiative correction to $\sin^2\theta_{\rm W}$ by the virtual Higgs boson in loops is

$$\delta_{\sin^2\theta_{\rm W}} = \frac{+5\alpha_{\rm em}}{24\pi} \log \frac{M_{\rm H}}{M_{\rm Z}} \ ,$$

and should be compared with the quadratic dependence on $m_{\rm t}$ of $\delta_{\sin^2\theta_{\rm W}}$ given in (1). This explains why low-energy observables are relatively insensitive to the Higgs boson mass and illustrates the difficulty in devising experiments that can probe the Higgs sector by virtual quantum loop effects. This also provides an important constraint on any model of symmetry breaking beyond the Standard Model.

17.3.3 Supersymmetry and Technicolor

The dichtomy of weakly coupled regime (small λ) and the strongly coupled regime (large λ) provides a framework for examining new physics beyond the Standard Model. Let us just mention two possible dynamical mechanisms of breaking the electroweak symmetry: Supersymmetry and technicolor which are respectively associated with the relatively light and heavy Higgs mass, i.e. the weak- and strong-coupling regimes of λ.

Supersymmetry. From general considerations, supersymmetry (SUSY) is the only nontrivial extension of the Lorentz group. The simplest extension, called $N = 1$ SUSY, requires the introduction of a single anticommuting degree of freedom to space-time. This implies, for example that a spin-$\frac{1}{2}$ field ψ is necessarily associated with a scalar spin-0 field ϕ. Thus, supersymmetry is a symmetry that links bosons and fermions. Why is SUSY relevant to the electroweak symmetry breaking? A quick answer is that SUSY may offer a framework for the scalar Higgs field to participate naturally in the weak interaction on the same footing as leptons and quarks. SUSY is particularly well suited to the weakly coupled regime ($\lambda < 1$). It may give rise to a mechanism of electroweak symmetry breaking associated with the top quark using the renormalization group evolution. It is beyond the scope of this book to explain this point. Let us only mention that in $N = 1$ SUSY, there are two complex Higgs field doublets. One of these doublets, related to the heaviness of the top quark, has an interacting potential unstable by the renormalization group evolution, such that μ^2 could be driven naturally to a negative value[3].

Technicolor. Technicolor is directly inspired by the following fact. In the standard QCD and electroweak interaction, the three initially massless gauge bosons A^i, associated with the generators of $\mathrm{SU(2)_L}$ and introduced in (9.42), already acquire a tiny mass equal to $gf_\pi/(2\sqrt{2}) \approx 31$ MeV via the pion considered as a Goldstone boson. This can be seen as follows.

QCD with two massless u and d quarks has a global $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ symmetry represented by the doublets $\mathrm{q_L} = (\mathrm{u_L}, \mathrm{d_L})$ and $\mathrm{q_R} = (\mathrm{u_R}, \mathrm{d_R})$ which can be independently rotated in their respective SU(2) spaces. These two $\mathrm{SU(2)_L}$ and $\mathrm{SU(2)_R}$ groups are linked by the pairing of q and $\overline{\mathrm{q}}$ in the vacuum so that the operator $\overline{\mathrm{q}}\mathrm{q}$ acquires a nonvanishing vacuum expectation value. The overall symmetry $\mathrm{SU(2)_V}$ corresponding to $\mathrm{L} \oplus \mathrm{R}$ is unbroken and gives the isospin symmetry of QCD. The other $\mathrm{SU(2)_A}$ (from $\mathrm{L} \ominus \mathrm{R}$) associated with the axial current $a^i_\mu = \overline{q}\gamma_\mu\gamma_5\tau^i q$ (τ^i are the three Pauli matrices) is spontaneously broken, resulting in three Goldstone bosons, or pions. The matrix element of the current a^i_μ between the pion and the vacuum is well known; it is

$$\left\langle 0 \,\middle|\, a^i_\mu \,\middle|\, \pi^j(k) \right\rangle = \mathrm{i}\frac{f_\pi}{\sqrt{2}}\, k_\mu \delta^{ij} \quad , \quad f_\pi \approx 131 \text{ MeV} .$$

Even without the Higgs mechanism, the massless boson $A^i(x)$, when coupled to the current $J^i_\mu = \frac{1}{2}(v^i_\mu - a^i_\mu)$ built up by the u and d quark fields, allows the creation of a pion with amplitude $\mathrm{i}g(-\frac{1}{2})(\mathrm{i}f_\pi/\sqrt{2}\, k_\mu)$, in which the factor $-\frac{1}{2}$ comes from the coefficient of a^i_μ in $J^i_\mu = \frac{1}{2}(v^i_\mu - a^i_\mu)$.

The contribution of the pion to the vacuum polarization $\Pi_{\mu\nu}(k)$ of the A^i boson as depicted in Fig. 17.6 has a singularity $1/k^2$ near $k^2 = 0$. The

[3] Ibañez, L. E. and Ross, G. G., in *Perspective on Higgs Physics* (ed. Kane, G.). World Scientific, Singapore 1992

residue at the $k^2 = 0$ pole is $\rho = [g\, f_\pi/2\sqrt{2}]^2$. Together with the conservation of the current J^i_μ, the vacuum polarization must satisfy $k^\mu \Pi_{\mu\nu}(k) = 0$, so that near the $k^2 = 0$ pole, the vacuum polarization $\Pi_{\mu\nu}(k)$ has the form

$$\Pi^{ij}_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \left[\frac{g f_\pi}{2\sqrt{2}}\right]^2 \delta^{ij} = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \rho\, \delta^{ij} \ .$$

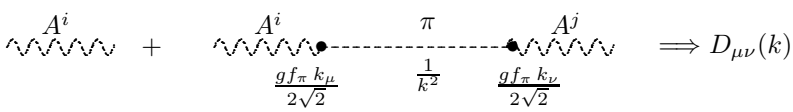

Fig. 17.6. The Goldstone pion of QCD gives mass to the gauge boson A^i

As in (15.9)–(15.11) and following the discussion below (15.11), the propagator of the A^i boson, dressed by the pion, may be written as

$$D_{\mu\nu}(k) = \frac{-\mathrm{i} g_{\mu\nu}}{k^2[1 - \rho/k^2]} \ .$$

It has a pole at $k^2 = \rho$, i.e. the massless A^i boson gets a mass $g f_\pi/2\sqrt{2}$ by absorbing the Goldstone pion coming from the spontaneous $\mathrm{SU(2)_A}$ symmetry breaking of QCD. So QCD already can give a mass $g f_\pi/2\sqrt{2} \approx 31$ MeV to the A^i boson, which may eventually emerge as the W boson.

Since the true W boson mass is $gv/2$, Susskind and Weinberg proposed technicolor as a copy of QCD scaled up by the factor $v\sqrt{2}/f_\pi \approx 2600$, with a techni-pion having a decay constant $\widetilde{F}_\pi = v$. This techni-pion is the Goldstone boson built up from $\widetilde{\mathrm{U}}$ and $\widetilde{\mathrm{D}}$ techni-quarks and would be responsible for the weak-boson masses.

Both SUSY and technicolor have rich spectra of new particles. Masses of techni-hadrons are expected in the TeV region, whereas some particles in SUSY may have masses in the range of a few hundred GeVs. The reader is referred to the very abundant literature on the subject (Further Reading).

Perspectives. Consistency of the Standard Model requires that the new physics responsible for mass generation may occur at an energy scale of about 1 TeV or less. The future high-energy colliders, in particular the LHC at CERN, are intended to explore this energy region. However, discoveries may also come from the lower-energy, high-precision, high-intensity physics in which heavy flavors, in particular the B meson, are important. Advances in particle physics may well lie in the least expected directions, but a study of the Standard Model suggests that we should address the following questions:

(i) Top quark physics: Why are all other fermions so much lighter? Does the top quark have something to do with the gauge symmetry breaking?

(ii) Neutrino masses and mixing: If neutrinos are truly neutral Majorana particles, there is at least one more possibility for the neutrinos to mix than

for the quarks (which are strictly Dirac particles). The Majorana neutrino masses can only be generated outside the Standard Model.

(iii) Nonstandard CP violation: Is there any other mechanism of CP violation than the KM one, where only charged currents are involved ?

Problems

17.1 Higgs boson in $e^+ + e^- \to W^+ + W^-$**.** How many tree diagrams are there for the above reaction? Show that, without the Higgs boson exchanged, the amplitude blows up as $s^{1/2}$ for the production of longitudinally polarized W + W, where $s^{1/2}$ is the total energy in the center-of-mass system.

17.2 Amplitude $W_L + W_L \to W_L + W_L$ **at high energy.** From the five diagrams of Fig. 17.5, write down the amplitude of the longitudinally polarized W–W scattering in the limit $s \gg M_H^2, M_W^2, M_Z^2$, and check (14).

17.3 H→ g+ g triangle loop. The amplitude of the Higgs field interacting with two gluons (Fig. 17.3) of momenta k_1, k_2 and polarizations $\varepsilon_\mu(k_1)$, $\varepsilon_\nu(k_2)$ has the following form

$$A(k_1, k_2)\,[k_2^\mu k_1^\nu - k_1 \cdot k_2 g^{\mu\nu}]\,\varepsilon_\mu(k_1)\,\varepsilon_\nu(k_2) \equiv I^{\mu\nu}\,\varepsilon_\mu(k_1)\,\varepsilon_\nu(k_2)$$

which satisfies the gauge invariance condition $(k_1)_\mu I^{\mu\nu} = (k_2)_\nu I^{\mu\nu} = 0$. The effective Higgs boson–gluon–gluon coupling is described by the coefficient $A(k_1, k_2)$. Compute $A(k_1, k_2)$ from the triangle diagram of Fig. 17.3 with only the internal top quark. One should recover the function $F_{1/2}$ in (6).

Suggestions for Further Reading

Production and decay of the Higgs boson:

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