11 Neutral K Mesons and CP Violation

The neutral K mesons, with their medium-sized masses and their capacity of interacting both weakly and strongly, seem to be specially selected by nature to demonstrate through a few typical phenomena the reality of quantum effects. Even if they did not exist, as L. B. Okun once said, we would have invented them in order to illustrate the fundamental principles of quantum physics.

Four of these phenomena will be studied in this chapter. The first to be considered arises from the existence, in the presence of strong interactions only, of two degenerate states of opposite strangeness quantum numbers, called K^0 and $\overline{\mathrm{K}}^0$. These states are mixed by the weak interaction, which does not conserve strangeness, to produce two states quite similar in their masses (which differ only by $\Delta m = 3.49 \times 10^{-6}\,\mathrm{eV} = 5.30 \times 10^{9}\,\mathrm{s}^{-1}$) but very dissimilar in their distinctive decay modes and their lifetimes. One short-lived, $\mathrm{K_S}$, with lifetime $\tau_\mathrm{S} = 8.92 \times 10^{-11}$ s, and the other long-lived, $\mathrm{K_L}$, with lifetime $\tau_\mathrm{L} = 5.17 \times 10^{-8}$ s. The existence of these states results in a second property called *strangeness oscillations*: a pure strangeness eigenstate, say K^0, produced at a given time becomes at a later time a mixture of K^0 and $\overline{\mathrm{K}}^0$. The amplitude of such a meson beam oscillates in time with a period of $T = 2\pi/\Delta m = 1.18 \times 10^{-9}$ s.

The third property to be studied concerns the *regeneration* of $\mathrm{K_S}$. Consider a beam of K^0 produced at time $t = 0$ by some strong interaction process. After a lapse of time of the order of τ_S, every K meson decays into two pions through its $\mathrm{K_S}$ components; but this process ceases to occur after $\tau_\mathrm{S} \ll t \ll \tau_\mathrm{L}$ when all the $\mathrm{K_S}$ have gone, leaving only the $\mathrm{K_L}$ in the beam. If a block of matter is now placed on the beam path, the $\mathrm{K_L}$ will interact with matter and partially transforms itself into $\mathrm{K_S}$.

Last but not least, we will discuss the CP violation by weak interactions, a phenomenon first reported in 1964 and to this day observed only in the neutral K meson system. It is one of the most fundamental but least understood properties of particle physics. To explore the physics of heavy flavors, several B meson factories are being constructed in the U. S. A. and Japan. Among the first projects to be carried out at these laboratories will certainly see the investigation of the nature, the origin, and the mechanism of the CP violation. The importance of this study may even have a larger impact if the enormous disproportion of matter (baryons) and antimatter (antibaryons) existing in the universe is regarded as a direct consequence of a CP violation that occurred just after the Big Bang.

11.1 The Two Neutral K Mesons

From Dirac's work (Chap. 3), we know that each particle corresponds to an antiparticle, both having equal masses, spins, and lifetimes. But their charges of all types (electric, leptonic, baryonic, flavor, or color) are equal in magnitudes but opposite in signs. Among the electrically neutral particles, the neutron is distinct from the antineutron, but certain particles such as the photon, the mesons π^0 and η are identical to their respective antiparticles. In contrast, the neutral K mesons have peculiarly mixed identities.

Recall first that the pseudoscalar mesons K^+, K^0 and their conjugates K^- and $\overline{K}^0$ are bound states, composed mainly of quarks u, d, and s. In Table 11.1, their quantum numbers (strangeness, isospin) and quark contents are given.

Table 11.1. Strange pseudoscalar mesons

K Mesons	Quark contents	I_3	Strangeness S
K^+	$\bar{s}u$	$+\frac{1}{2}$	$+1$
K^0	$\bar{s}d$	$-\frac{1}{2}$	$+1$
$\overline{K}^0$	$\bar{d}s$	$+\frac{1}{2}$	-1
K^-	$\bar{u}s$	$-\frac{1}{2}$	-1

The two mesons $\overline{K}^0$ and K^0 are quite distinct in the presence of strong interactions which conserve strangeness. Consider for example the strong production process $\pi^- + p \to K^0 + \Lambda$. The initial state has zero strangeness; therefore, since $\Lambda \equiv sdu$ has strangeness $S = -1$, it is a K^0 with $S = +1$ that is produced in the final state, and not a $\overline{K}^0$. If strange particles are produced in a strong interaction (normally from nonstrange particles), they are always produced in pairs of particles of opposite strangeness quantum numbers so as to conserve total strangeness (a phenomenon called the *associated production*). On the other hand, since the baryonic number is equally conserved by the strong interaction and since the initial state (the proton) has baryonic number $N_B = 1$ in the present example, the final state cannot be $\overline{K}^0 + \overline{\Lambda}$ in spite of its correct strangeness. Thus, from the point of view of the strong interaction, K^0 is as distinct from $\overline{K}^0$ as the neutron is from the antineutron.

The difference between these two pairs K^0–$\overline{K}^0$ and n–$\bar{n}$ appears in the presence of the weak interaction. As far as we know, the baryonic number is conserved in all situations (the proton lifetime is greater than 10^{39} s), but strangeness is not, being broken in weak processes. The conservation of N_B forbids transitions between the neutron and the antineutron because there exist no common intermediate states connecting those two states. On the other hand, both $\overline{\mathrm{K}}^0$ and K^0 can decay into pions via strangeness-violating weak transitions. Thus, the transmutation of K^0 into $\overline{\mathrm{K}}^0$, or inversely of $\overline{\mathrm{K}}^0$ into K^0, can proceed through common intermediates states of pions, as in

$$\mathrm{K}^0 \longrightarrow (2\pi,\ 3\pi) \longrightarrow \overline{\mathrm{K}}^0 . \tag{11.1}$$

On the quark level, the transitions $\mathrm{K}^0 \leftrightarrow \overline{\mathrm{K}}^0$ are represented by the Feynman box diagrams in Fig. 11.1. Via the weak interaction, the $\overline{\mathrm{s}}$ and d quarks of K^0 annihilate into a pair $\mathrm{W}^+\mathrm{W}^-$ (Fig. 11.1a) or a pair of quark–antiquark $\mathrm{Q}_i\overline{\mathrm{Q}}_j$ in all possible combinations of the three $\mathrm{u, c, t}$ quarks (Fig. 11.1b). These pairs $\mathrm{W}^+\mathrm{W}^-$ and $\mathrm{Q}_i\overline{\mathrm{Q}}_j$ then transform by the same weak interaction into s and $\overline{\mathrm{d}}$, giving a $\overline{\mathrm{K}}^0$ in the final state. Since these transitions change strangeness by two units ($|\Delta S| = 2$), they must proceed through the weak interaction.

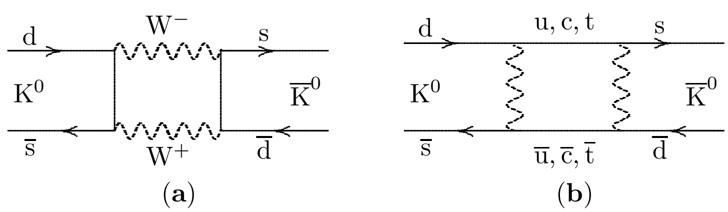

Fig. 11.1 a, b. $\mathrm{K}^0 \leftrightarrow \overline{\mathrm{K}}^0$ transition through $\overline{\mathrm{s}}\,\mathrm{d} \to \overline{\mathrm{d}}\,\mathrm{s}$

As implicitly understood in the above discussion, K^0 and $\overline{\mathrm{K}}^0$ are respectively strangeness eigenstates with eigenvalues $S = 1$ and $S = -1$, a meaningful statement only in the absence of weak interactions. Since S is not conserved by weak interactions, another quantum number has to be used to classify the neutral K mesons wherever the weak interaction operates. It turns out that the combined charge conjugation and parity operators, $\mathcal{CP}$, provide an almost perfect candidate for this role, 'almost' because the symmetry they represent is in fact only slightly violated (by a factor of 10^{-3}), while parity $\mathcal{P}$ and charge conjugation $\mathcal{C}$ are each separately violated at the highest level by weak interactions.

Leaving temporarily aside the question of its violation, we assume in this and the next three sections that CP is a good symmetry even in the presence of the weak interaction. As the K mesons are pseudoscalar, $\mathcal{P}\left|\overline{\mathrm{K}}^0\right\rangle = -\left|\overline{\mathrm{K}}^0\right\rangle$ and $\mathcal{P}\left|\mathrm{K}^0\right\rangle = -\left|\mathrm{K}^0\right\rangle$, and as $\overline{\mathrm{K}}^0$ and K^0 are charge conjugates to each other, $\mathcal{C}\left|\mathrm{K}^0\right\rangle = \left|\overline{\mathrm{K}}^0\right\rangle$ by an appropriate choice of phase, one gets

$$\mathcal{CP}\left|\mathrm{K}^0\right\rangle = -\left|\overline{\mathrm{K}}^0\right\rangle \quad , \quad \mathcal{CP}\left|\overline{\mathrm{K}}^0\right\rangle = -\left|\mathrm{K}^0\right\rangle . \tag{11.2}$$

From this it follows that the orthogonal linear combinations

$$\left|K_1^0\right\rangle \equiv \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\right) \quad , \quad \left|K_2^0\right\rangle \equiv \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\right) \; , \tag{11.3}$$

are eigenstates of CP :

$$\mathcal{CP}\left|K_1^0\right\rangle = +\left|K_1^0\right\rangle \quad , \quad \mathcal{CP}\left|K_2^0\right\rangle = -\left|K_2^0\right\rangle . \tag{11.4}$$

The state K_1^0 is even and K_2^0 is odd under CP.

To determine their decay modes into pions, it suffices to find the corresponding CP-parities of the multipion states. It turns out (Problem 11.1) that neutral two-pion states ($\pi^0+\pi^0$ and $\pi^+ + \pi^-$) are CP-even and neutral three-pion states ($\pi^0+\pi^0+\pi^0$ and $\pi^0+\pi^++\pi^-$) are CP-odd. Since CP is assumed to be conserved, the only allowed decay modes are

$$K_1^0 \to 2\pi \;, \qquad K_2^0 \to 3\pi . \tag{11.5}$$

It is noteworthy that these are the only hadronic decay channels open to the neutral K mesons, a fact attributed to the particular value of their mass, just slightly over three times the mass of the pion. This fact also implies that kinematics favors the two-pion mode over the three-pion mode because the kinetic energy available to the reaction products is larger, and the phase space is correspondingly larger in the former than in the latter. Therefore, K_1^0 has a proportionally larger decay width or equivalently a shorter lifetime than K_2^0.

This was a result predicted by Gell-Mann and Pais and later confirmed by experiment. What was observed was that the neutral K mesons decayed in two different hadronic channels at two different time scales. The first type goes through two-pion channels, with lifetime $\tau_S = 8.92\times10^{-11}$ s, and is called K_S. The second type, which can decay into three pions with characteristic time $\tau_L = 5.17\times10^{-8}$ s, is called K_L. Assuming CP conservation, one may identify K_S with the CP-even state K_1^0, and K_L with the CP-odd state K_2^0.

This double property, mass degeneracy and distinct lifetimes, is unique to the neutral K mesons. The situation is completely different with the neutral D mesons and the neutral B mesons, which yet parallel the K mesons in their quark compositions only with heavier flavors.

11.2 Strangeness Oscillations

We now consider the evolution in time of the amplitudes of states K_S and K_L. They are of the general form

$$a(t) = a(0)\exp[-\mathrm{i}(E - \tfrac{\mathrm{i}}{2}\Gamma)t] \, , \tag{11.6}$$

here E is the energy of the state and Γ its total decay width. The term $\mathrm{i}\Gamma/2$ is what is needed to yield the familiar exponential decay law

$$I(t) = a(t)a(t)^* = |a(0)|^2\mathrm{e}^{-\Gamma t} \, , \tag{11.7}$$

which says that the particle decays at the rate given by Γ. For a stable noninteracting particle, $\Gamma = 0$ and the amplitude $a(t)$ is given by the usual $\exp(-\mathrm{i}p \cdot x)$. In the particle rest frame, the particle energy equals its mass $E = m$, and its lifetime is given by $\tau = \hbar/\Gamma$.

Thus, the amplitudes that describe the time evolution of $\mathrm{K_S}$ and $\mathrm{K_L}$ are respectively

$$a_{\mathrm{S}}(t) = a_{\mathrm{S}}(0)\mathrm{e}^{-(\frac{\Gamma_{\mathrm{S}}}{2} + \mathrm{i}m_{\mathrm{S}})t} \quad , \quad a_{\mathrm{L}}(t) = a_{\mathrm{L}}(0)\mathrm{e}^{-(\frac{\Gamma_{\mathrm{L}}}{2} + \mathrm{i}m_{\mathrm{L}})t} . \tag{11.8}$$

There is no reason to expect here that the corresponding masses m_{S} and m_{L} be equal even though by CPT invariance the masses of $\overline{\mathrm{K}}^0$ and K^0 must be identical (Chap. 5). However, as the decay modes and the lifetimes of $\mathrm{K_S}$ and $\mathrm{K_L}$ are bound to differ from effects of the $|\Delta S| = 2$ effective interactions mentioned above, it is expected that $\Delta m \equiv m_{\mathrm{L}} - m_{\mathrm{S}} \neq 0$. We will now discuss how Δm is calculated and measured, and how it gives rise to the oscillation phenomenon observed in neutral K meson beams.

Suppose at $t = 0$ a K^0 beam is produced, e.g. by the strong production process $\pi^- + \mathrm{p} \to \mathrm{K}^0 + \Lambda$. Its amplitude written in terms of $\mathrm{K_S} = \mathrm{K}_1^0$ and $\mathrm{K_L} = \mathrm{K}_2^0$ is $\mathrm{K}^0 = \frac{1}{\sqrt{2}}(\mathrm{K_S} + \mathrm{K_L})$, or $a_{\mathrm{K}^0}(t) = \frac{1}{\sqrt{2}}\left[a_{\mathrm{S}}(t) + a_{\mathrm{L}}(t)\right]$. The beam intensity for K^0, call it $I_0(t)$, is given by

$$I_0(t) = \frac{1}{2}\left[a_{\mathrm{S}}(t) + a_{\mathrm{L}}(t)\right]\left[a_{\mathrm{S}}(t) + a_{\mathrm{L}}(t)\right]^* , \tag{11.9}$$

or equivalently, making use of (8), the normalized density $N(t) = I(t)/I(0)$ is given by

$$N_0(t) = \frac{1}{4}\Big[\mathrm{e}^{-\Gamma_{\mathrm{S}}t} + e^{-\Gamma_{\mathrm{L}}t} + 2\,\mathrm{e}^{-(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})\frac{t}{2}}\cos(\Delta m\, t)\Big] . \tag{11.10}$$

Similarly, we may write the normalized density for the $\overline{\mathrm{K}}^0$ beam (produced at $t = 0$ for example by the reaction $\pi^+ + \mathrm{p} \to \overline{\mathrm{K}}^0 + \mathrm{K}^+ + \mathrm{p}$) as

$$N_{\bar{0}}(t) = \frac{1}{4}\Big[\mathrm{e}^{-\Gamma_{\mathrm{S}}t} + e^{-\Gamma_{\mathrm{L}}t} - 2\,\mathrm{e}^{-(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})\frac{t}{2}}\cos(\Delta m\, t)\Big] . \tag{11.11}$$

Thus, the two beams oscillate with frequency $\Delta m/2\pi$ (Fig. 11.2). Since $\Delta m\,\tau_{\mathrm{S}} = 0.47$, the oscillation waves will be clearly visible at t of the order of a few τ_{S}, before all the $\mathrm{K_S}$ have died out, leaving only the $\mathrm{K_L}$ in the beam. As $c\tau_{\mathrm{S}} = 2.67$ cm and $c\tau_{\mathrm{L}} = 1551$ cm, the $\mathrm{K_L}$ will survive long after all the $\mathrm{K_S}$ have gone. In a beam made up entirely of K^0 at time $t = 0$, mesons $\overline{\mathrm{K}}^0$ will appear far from the production source through their presence in $\mathrm{K_L}$ with equal probability as K^0. Similarly, an initially pure $\overline{\mathrm{K}}^0$ will progressively become contaminated with K^0.

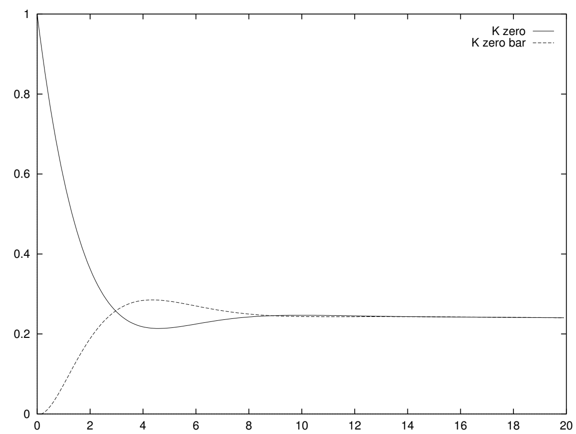

Fig. 11.2. Plots of $N_0(t)$ and $N_{\bar{0}}(t)$, t in units of 10^{-10} s

The oscillations can be detected from observing the $\mathrm{K}_{\ell 3}$ decay modes. In fact, from the quark contents of $\mathrm{K}^0(\bar{\mathrm{s}}\mathrm{d})$ and $\overline{\mathrm{K}}^0(\mathrm{s}\bar{\mathrm{d}})$, it follows that $\bar{\mathrm{s}} \to \bar{\mathrm{u}}+\ell^+ +\nu_\ell$ ($\mathrm{s} \to \mathrm{u}+\ell^- +\overline{\nu}_\ell$) and therefore the $\overline{\mathrm{K}}^0$ and K^0 decays are governed by the $\Delta S = \Delta Q$ rule:

$$\mathrm{K}^0 \to \pi^- + \ell^+ + \nu_l \quad , \quad \overline{\mathrm{K}}^0 \to \pi^+ + \ell^- + \bar{\nu}_l \, . \tag{11.12}$$

Indeed from the initial K^0 to the final π^- hadronic states, there is a change in strangeness, $\Delta S = (+1)-(0) = +1$, and in electric charge $\Delta Q = (0)-(-1) = +1$, so $\Delta S = \Delta Q$. The same change $\Delta S = \Delta Q = -1$ occurs in $\overline{\mathrm{K}}^0 \to \pi^+$. Then from this rule, we can *identify* K^0 by its decay product e^+, and the $\overline{\mathrm{K}}^0$ by its e^-. By recording the number of the emitted positrons (N^+) and electrons (N^-), one can measure the charge asymmetry

$$\delta(t) \equiv \frac{N^+ - N^-}{N^+ + N^-} = \frac{N_0(t) - N_{\bar{0}}(t)}{N_0(t) + N_{\bar{0}}(t)} \tag{11.13}$$

whose time evolution as given by (10) and (11) is

$$\delta(t) \sim 2\,\mathrm{e}^{-(\Gamma_S+\Gamma_L)\frac{t}{2}} \, \cos(\Delta m\, t)\, . \tag{11.14}$$

This asymmetry displays a sinusoidal time oscillation, from which one may infer the magnitude of the mass difference (Fig. 11.3). Experiment gives the mass difference $|\Delta m| \simeq 3.49 \times 10^{-6}\,\mathrm{eV} \simeq 5.30 \times 10^9\,\mathrm{s}^{-1}$ (the sign of Δm will be considered in the following section).

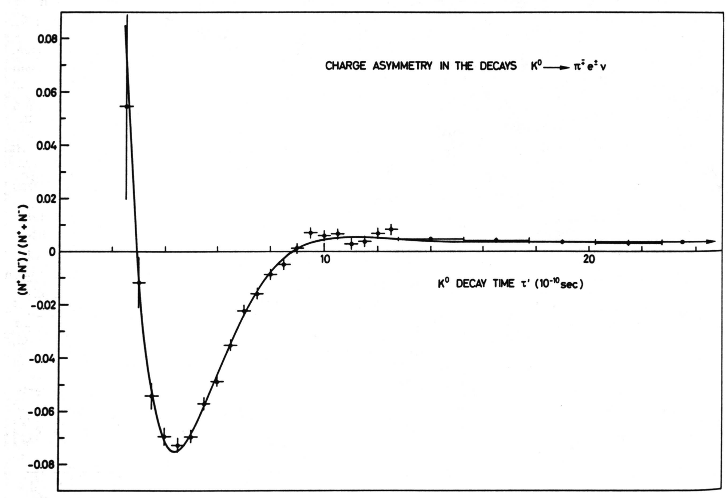

Fig. 11.3. The charge asymmetry $\delta(t)$ in the decays $K^0 \to \pi^{\mp}e^{\pm} + \nu$, as measured by Gjesdal, S. et al., Phys. Lett. **52B** (1974) 113. Reprinted by permission of Elsevier Science

11.3 Regeneration of K_S^0

Let us consider again a K^0 beam produced at $t = 0$, and suppose that, long after the vanishing of all the K_S from the beam so that only the K_L component remains, we place on the path of the beam a block of matter which may be regarded for all practical purposes as composed of protons and neutrons. Pais and Piccioni suggested in 1955 that, as the K^0 and $\overline{K}^0$ components present in K_L interact very differently with matter, the K_S would eventually reappear in the beam. That this is indeed the case can be seen by examining the hadronic interactions of K^0 and $\overline{K}^0$ with the neutron and the proton. While there are no differences in their elastic scatterings, only $\overline{K}^0$ can be absorbed by matter through $\overline{K}^0 + p \to \Lambda + \pi^+$ or $\overline{K}^0 + n \to \Lambda + \pi^0$. Similar processes for K^0 are forbidden by the strangeness conservation: thus $K^0 + p \not\to \Lambda + \pi^+$ and $K^0 + n \not\to \Lambda + \pi^0$. This difference in the absorption properties of K mesons by matter is essential to understanding the regeneration of K_S as the beam is traveling through matter. If we call f and $\bar{f}$ the amplitudes of scattering of K^0 and $\overline{K}^0$ on the atomic nuclei in matter, then, as we have just seen, $f \neq \bar{f}$. Now, the wave function that enters the block is that of K_L:

$$\psi_i = \psi_{K_L} = \frac{K^0 + \overline{K}^0}{\sqrt{2}} . \tag{11.15}$$

Interactions of the mesons with matter will cause the wave function to change, so that at exit it is given by

$$\psi_f = \frac{1}{\sqrt{2}}(f\,\mathrm{K}^0 + \overline{f}\,\overline{\mathrm{K}}^0) = \frac{1}{2}[(f+\overline{f})\,\mathrm{K_L} + (f-\overline{f})\mathrm{K_S}]\,. \tag{11.16}$$

In other words the $\mathrm{K_L}$ amplitude will become $\mathrm{K_L} + r\mathrm{K_S}$, where $r = \frac{f-\bar{f}}{f+f}$ parameterizes the regeneration process, a factor which depends on the properties of the sample of matter that the beam has gone through. Since $r \neq 0$, there must be regeneration, and the phenomenon has been observed.

The $\mathrm{K_S}$ regeneration can be exploited to measure the mass difference Δm, a quantity of key importance in the studies of neutral K mesons. For this purpose, let us place on the path of the beam two blocks of matter separated by a distance d that may be varied at will. When $\mathrm{K_L}$ and the regenerated $\mathrm{K_S}$ go through the second block, their oscillations interfere in a manner different than in the vacuum and that depends on d. By letting d vary, one may determine the sign of Δm from the observed interference effects between the $\mathrm{K_L}$ and $\mathrm{K_S}$. It turns out that $\Delta m > 0$, that is, $m_\mathrm{L} > m_\mathrm{S}$. The $\mathrm{K_S}$ regeneration phenomenon, which arises from an interplay between strangeness hadronic eigenstates and weakly interacting CP eigenstates, displays many similarities with the oscillations of neutrinos in matter, which we will discuss in the next chapter.

The two pairs $(\mathrm{K}^0, \overline{\mathrm{K}}^0)$ and $(\mathrm{K_L}, \mathrm{K_S})$, which represent respectively the eigenstates of strangeness S and discrete symmetry CP, are *dual* from the quantum physics viewpoint. The neutral K system which decays into $\pi^-\mathrm{e}^+\nu_\mathrm{e}$ and $\pi^+\mathrm{e}^-\overline{\nu}_\mathrm{e}$ are respectively K^0 and $\overline{\mathrm{K}}^0$. These modes involve the strangeness $\mathcal{S}$ operator. The same neutral K system which decays into two-pion and three-pion modes are respectively $\mathrm{K_S}$ and $\mathrm{K_L}$. These decays concern the $\mathcal{CP}$ operator. The semileptonic and hadronic decay modes are used to select the eigenstates of $\mathcal{S}$ and $\mathcal{CP}$ respectively.

In some sense, the pairs $(\mathrm{K}^0, \overline{\mathrm{K}}^0)$ and $(\mathrm{K_L}, \mathrm{K_S})$ are similar to the spin states σ_x and σ_y of the electron. The operators $\mathcal{S}$ and $\mathcal{CP}$ play the role of two orthogonal magnetic fields H_x and H_y which project out the spins σ_x and σ_y. According to the Heisenberg uncertainty principle and demonstrated by the Stern–Gerlach experiment, it is impossible to quantize simultaneously the components σ_x and σ_y of the electron spin since they do not commute. Similarly, the two pairs $(\mathrm{K}^0, \overline{\mathrm{K}}^0)$ and $(\mathrm{K_L}, \mathrm{K_S})$ cannot be simultaneously determined, the operators $\mathcal{S}$ and $\mathcal{CP}$ also do not commute in weak interaction. We have a simple illustration of the two familiar concepts in quantum physics, state superposition and quantization.

11.4 Calculation of Δm

All three phenomena that we have just described depend on the mass difference between $\mathrm{K_L}$ and $\mathrm{K_S}$. It would therefore be interesting to see how Δm is viewed in the standard model. Let us write m_L and m_S as the real part of the expectation values of a certain Hamiltonian operator $H = H^{(0)} + H^{(2)}$, the imaginary part is related to their decay widths (the subscripts 0 and 2 correspond to the $\Delta S = 0$ and $|\Delta S| = 2$ transitions):

$$\begin{aligned} m_L &\equiv \mathrm{Re}\,[\langle \mathrm{K_L} \,|\, H \,|\, \mathrm{K_L}\rangle] = \mathrm{Re}\left[\frac{1}{2}\left\langle \mathrm{K}^0 + \overline{\mathrm{K}}^0 \,|\, H \,|\, \mathrm{K}^0 + \overline{\mathrm{K}}^0 \right\rangle\right] \;, \\ m_S &\equiv \mathrm{Re}\,[\langle \mathrm{K_S} \,|\, H \,|\, \mathrm{K_S}\rangle] = \mathrm{Re}\left[\frac{1}{2}\left\langle \mathrm{K}^0 - \overline{\mathrm{K}}^0 \,|\, H \,|\, \mathrm{K}^0 - \overline{\mathrm{K}}^0 \right\rangle\right] \;, \end{aligned} \tag{11.17}$$

from which is deduced

$$\Delta m \equiv m_\mathrm{L} - m_\mathrm{S} = \mathrm{Re}\left[\left\langle \mathrm{K}^0 \,\middle|\, H^{(2)} \,\middle|\, \overline{\mathrm{K}}^0 \right\rangle + \left\langle \overline{\mathrm{K}}^0 \,\middle|\, H^{(2)} \,\middle|\, \mathrm{K}^0 \right\rangle\right] \;. \tag{11.18}$$

This relation tells us that Δm stems from the effective $|\Delta S| = 2$ interaction $H^{(2)}$ that causes the transitions between K^0 and $\overline{\mathrm{K}}^0$, represented by the diagrams in Fig. 11.1. To illustrate the calculation of the matrix elements involved, at first we keep only the contributions from the u quark in the intermediate states and assume that the four-momenta of the external quarks may be neglected. Applying the Feynman rules for the standard model (Chap. 9), we obtain for the process represented in Fig. 11.1a the transition amplitude

$$\begin{aligned} \mathcal{M}_a = &\mathrm{i}\left[\frac{-\mathrm{i}g}{2\sqrt{2}}V^*_\mathrm{ud}\right]^2\left[\frac{-\mathrm{i}g}{2\sqrt{2}}V_\mathrm{us}\right]^2\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\bar{u}(\mathrm{s})\gamma_\lambda(1-\gamma_5)\frac{\mathrm{i}(\not{k}+m_\mathrm{u})}{k^2-m_\mathrm{u}^2}\gamma_\rho(1-\gamma_5)v(\mathrm{d}) \\ &\times \bar{v}(\mathrm{s})\gamma_\alpha(1-\gamma_5)\frac{\mathrm{i}(\not{k}+m_\mathrm{u})}{k^2-m_\mathrm{u}^2}\gamma_\sigma(1-\gamma_5)u(\mathrm{d})\,\frac{-\mathrm{i}g^{\lambda\sigma}}{k^2-M_\mathrm{W}^2}\;\frac{-\mathrm{i}g^{\alpha\rho}}{k^2-M_\mathrm{W}^2}\,. \end{aligned} \tag{11.19}$$

The Feynman–'t Hooft ($\xi = 1$) gauge is used for the W-boson propagator. Of course, the end result [(see (36) below] should be independent of the particular gauge used in the calculation, and (19) can be written as

$$\mathcal{M}_a = \frac{\mathrm{i}g^4(V^*_\mathrm{ud}V_\mathrm{us})^2}{16}I_{\mu\nu}T^{\mu\nu} \;, \quad \text{with} \tag{11.20}$$

$$\begin{aligned} I_{\mu\nu} &= \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(k^2-M_\mathrm{W}^2)^2(k^2-m_\mathrm{u}^2)^2} \;, \qquad \text{and} \\ T^{\mu\nu} &= [\bar{u}(\mathrm{s})\gamma_\lambda\gamma^\mu\gamma_\rho(1-\gamma_5)v(\mathrm{d})]\,[\bar{v}(\mathrm{s})\gamma^\rho\gamma^\nu\gamma^\lambda(1-\gamma_5)u(\mathrm{d})]\,. \end{aligned} \tag{11.21}$$

Neglecting $m_\mathrm{u}^2/M_\mathrm{W}^2$ and using the integral formulas in the Appendix, the integral $I_{\mu\nu}$ can be easily performed, leading to the result

$$I_{\mu\nu} = -\mathrm{i}\,\frac{g_{\mu\nu}}{64\pi^2M_\mathrm{W}^2} \;. \tag{11.22}$$

With the help of the identity

$$\gamma_\lambda\gamma_\mu\gamma_\rho = g_{\lambda\mu}\gamma_\rho + g_{\mu\rho}\gamma_\lambda - g_{\lambda\rho}\gamma_\mu + \mathrm{i}\epsilon_{\lambda\mu\rho\alpha}\gamma_5\gamma^\alpha \;, \tag{11.23}$$

one gets

$$g_{\mu\nu}T^{\mu\nu} = 4\ [\bar{u}(\mathrm{s})\gamma_\lambda(1-\gamma_5)v(\mathrm{d})]\ [\bar{v}(\mathrm{s})\gamma^\lambda(1-\gamma_5)u(\mathrm{d})] \\ \Longrightarrow 4\ \Theta^{|\Delta S|=2}\ , \tag{11.24}$$

where $\Theta^{|\Delta S|=2} \equiv [\bar{s}\gamma_\lambda(1-\gamma_5)d]\ [\bar{s}\gamma^\lambda(1-\gamma_5)d]$.

The spinors u and v have been replaced by the corresponding quark fields, so that an effective operator on the Hilbert space, $H_a^{|\Delta S|=2}$, may be obtained from the transition amplitude. Similarly, one may calculate the effective operator corresponding to the process represented in Fig. 11.1b. It turns out to be exactly equal to $H_a^{|\Delta S|=2}$. Putting together (20), (22), and (24), the full interaction operator with only u quark in the intermediate states is simply

$$H^{(2)} = 2H_a^{|\Delta S|=2} = \frac{G_\mathrm{F}^2}{4\pi^2}(V_\mathrm{ud}^* V_\mathrm{us})^2 M_\mathrm{W}^2\ \Theta^{|\Delta S|=2}\ , \tag{11.25}$$

where $g^2/8M_\mathrm{W}^2 = G_\mathrm{F}/\sqrt{2}$ is used.

The calculation of Δm then boils down to the calculation of the matrix element of $\Theta^{|\Delta S|=2}$ between states K^0 and $\overline{\mathrm{K}}^0$. We will calculate it in the *vacuum insertion approximation*, which implies keeping the vacuum as the only intermediate state. First, using Fierz's rearrangement (Appendix), the operator $\Theta^{|\Delta S|=2}$ can also be written as

$$\bar{s}_a\gamma_\mu(1-\gamma_5)d_a\ \bar{s}_b\gamma^\mu(1-\gamma_5)d_b = \bar{s}_a\gamma_\mu(1-\gamma_5)d_b\ \bar{s}_b\gamma^\mu(1-\gamma_5)d_a\ , \tag{11.26}$$

where $a, b = 1, 2, 3$ label the quark colors. The minus sign in the Fierz's transformation combined with the anticommutation rules for the fermionic field operators yields a positive sign in (26). Now we insert $|0\rangle\langle 0|$ between the two bilinear spinor products in all possible ways and define

$$\left\langle 0\,|\,A_{ab}^\mu\,|\,\mathrm{K}^0\right\rangle \equiv \left\langle 0\,|\,\bar{s}_b\gamma^\mu\gamma_5 d_a\,|\,\mathrm{K}^0\right\rangle = \left\langle 0\,|\,\bar{s}_b\gamma^\mu\gamma_5 u_a\,|\,\mathrm{K}^+\right\rangle = \frac{\mathrm{i}f_\mathrm{K}\,q^\mu}{\sqrt{2m_\mathrm{K}}}\frac{\delta_{ab}}{3}\ , \tag{11.27}$$

where the decay constant of the K meson, $f_\mathrm{K} \approx 160$ MeV, is extracted from the $\mathrm{K}^+ \to \mu^+ + \nu_\mu$ rate, like $f_\pi \approx 131$ MeV from $\pi^+ \to \mu^+ + \nu_\mu$ (Chap. 10). The denominator $\sqrt{2m_\mathrm{K}}$ comes from the K meson one-particle state normalization. We obtain in the vacuum insertion approximation

$$\left\langle \mathrm{K}^0 \,\middle|\, \Theta^{|\Delta S|=2} \,\middle|\, \overline{\mathrm{K}}^0 \right\rangle = \frac{2}{3}\,\frac{f_\mathrm{K}^2 m_\mathrm{K}^2}{2m_\mathrm{K}}\ . \tag{11.28}$$

In (28), the factor $\frac{2}{3} = \frac{1}{2}(1+\frac{1}{3})$ comes from the rearrangement of the color indices by using (26) and (27).

If the vacuum insertion approximation is relaxed, the result must be modified, but the necessary changes may be simply parameterized by a multiplicative B factor, so that (28) may be replaced by

$$\left\langle \mathrm{K}^0 \,\middle|\, \Theta^{|\Delta S|=2} \,\middle|\, \overline{\mathrm{K}}^0 \right\rangle = \frac{2}{3}\,\frac{f_\mathrm{K}^2 m_\mathrm{K}^2}{2m_\mathrm{K}}\,B\,, \tag{11.29}$$

$B = 1$ corresponds to the vacuum insertion (28). In a wide variety of models and in lattice gauge calculations, estimates for B differ from 1, within a factor of two. From (18), (25), and (29) we obtain the expression for Δm:

$$\Delta m = \mathrm{Re}\left\{ \frac{G_F^2}{6\pi^2}(V_\mathrm{ud}^* V_\mathrm{us})^2\, f_\mathrm{K}^2 m_\mathrm{K} M_\mathrm{W}^2 \right\}\, B\,. \tag{11.30}$$

The resulting value for Δm is larger than the measured value by a factor of 3×10^3. Clearly, something is fundamentally wrong, not so much with letting $B \approx 1$ as with neglecting the presence of other quarks, and especially of the c quark in the intermediate states. This oversight can be easily amended.

From the unitarity of the CKM matrix (Chap. 9), the matrix elements that are relevant to our calculation satisfy the relation

$$V_\mathrm{ud}^* V_\mathrm{us} + V_\mathrm{cd}^* V_\mathrm{cs} + V_\mathrm{td}^* V_\mathrm{ts} = 0\,. \tag{11.31}$$

Neglecting the last term, which is actually very small, being of the order of 10^{-4}, this relation yields

$$V_\mathrm{ud}^* V_\mathrm{us} = -V_\mathrm{cd}^* V_\mathrm{cs}\,. \tag{11.32}$$

This equation indicates there is a destructive interference between the u and c quarks in the transitions under consideration, an effect that will drastically reduce Δm to the level actually observed. It is precisely this interference effect between the u and c quarks in their actions in weak processes that underlies the GIM mechanism (Chap. 9). With (32), keeping both u and c quarks in the intermediate states has the effect of replacing in (19) the factor

$$\left[\frac{1}{k^2 - m_\mathrm{u}^2}\right]^2 \quad \text{by} \quad \left[\frac{1}{k^2 - m_\mathrm{u}^2} - \frac{1}{k^2 - m_\mathrm{c}^2}\right]^2 .$$

Therefore, the old expression (22) for the integral $I_{\mu\nu}$ is now replaced with

$$I_{\mu\nu}^{(\mathrm{u,c})} \equiv \int \frac{d^4k}{(2\pi)^4}\,\frac{k_\mu k_\nu (m_\mathrm{c}^2 - m_\mathrm{u}^2)^2}{(k^2 - m_\mathrm{u}^2)^2 (k^2 - m_\mathrm{c}^2)^2 (k^2 - M_\mathrm{W}^2)^2}\,. \tag{11.33}$$

$$\text{With }\; m_\mathrm{u} \ll m_\mathrm{c} \ll M_\mathrm{W}\,,\quad I_{\mu\nu}^{(\mathrm{u,c})} = \frac{-\mathrm{i}\, g_{\mu\nu}}{64\pi^2}\,\frac{m_\mathrm{c}^2 - m_\mathrm{u}^2}{M_\mathrm{W}^4}\,, \tag{11.34}$$

from which we finally get

$$\Delta m = \mathrm{Re}\left\{\frac{G_F^2}{6\pi^2}(V_{\mathrm{cd}}^* V_{\mathrm{cs}})^2 f_{\mathrm{K}}^2 m_{\mathrm{K}}\,(m_{\mathrm{c}}^2 - m_{\mathrm{u}}^2)\right\} B\,. \tag{11.35}$$

It is the GIM mechanism that allows replacing M_{W}^2 in (30) with $m_{\mathrm{c}}^2 - m_{\mathrm{u}}^2$ in (35), making possible a correct prediction for Δm, with $m_{\mathrm{c}} \approx 1.5$ GeV. Historically, Gaillard and Lee exploited this mechanism to predict the mass of the c quark before the discovery of the charmonium J/ψ in 1974. Let us also note that if u and c quarks are mass-degenerate, $m_{\mathrm{u}} = m_{\mathrm{c}}$, the suppression effect of the GIM mechanism would be complete, and Δm would vanish and there would be no transitions between K^0 and $\overline{\mathrm{K}}^0$ at all. This suppression can be easily understood in the framework of the standard model of the electroweak interaction. We saw in Chap. 9 that if the $Q = 2/3$ quarks (u, c, and t), or alternatively the $Q = -1/3$ quarks (d, s, and b), were *mass-degenerate*, there would be no need for a CKM mixing matrix, and there would be no flavor mixing between quarks having the same charge. More precisely, in that case each up-type quark would couple only to its own down-type weak-isospin partner by the charged currents, so that cross-family couplings would be completely absent, i.e. $V_{\mathrm{us}} = V_{\mathrm{cd}} = 0$.

As already mentioned, the GIM mechanism, which was invented primarily in order to suppress the flavor-changing neutral currents at the tree level, is still potent in higher-order loop diagrams. Processes depicted by the box diagrams in Fig. 11.1 are examples of its power.

Finally, let us examine the contributions of the top quark to Δm. A rough estimate using (35), with parameters for t replacing those for c, shows that a top quark mass of about 180 GeV, large as it is, cannot compensate for the smallness of the CKM matrix elements $V_{\mathrm{td}}^* V_{\mathrm{ts}}$ to yield a significant contribution to Δm. The exact unitarity of V_{CKM} and a more careful calculation in the R_ξ gauge, keeping $x_{\mathrm{c}} \equiv m_{\mathrm{c}}^2/M_{\mathrm{W}}^2$, $x_{\mathrm{t}} \equiv m_{\mathrm{t}}^2/M_{\mathrm{W}}^2$, lead to

$$\begin{aligned}\Delta m = \mathrm{Re}\Bigg\{\frac{G_{\mathrm{F}}^2}{6\pi^2} f_{\mathrm{K}}^2 m_{\mathrm{K}} m_{\mathrm{c}}^2 \Big[&(V_{\mathrm{cd}}^* V_{\mathrm{cs}})^2 g(x_c) + (V_{\mathrm{td}}^* V_{\mathrm{ts}})^2 \frac{x_t}{x_c} g(x_t) \\ &+2\,(V_{\mathrm{td}}^* V_{\mathrm{ts}} V_{\mathrm{cd}}^* V_{\mathrm{cs}})\, h(x_c, x_t)\Big]\Bigg\} B\,,\end{aligned} \tag{11.36}$$

where the functions $g(x)$ and $h(x,y)$ are given by[1]

$$\begin{aligned} g(x) &= \frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2} - \frac{3}{2}\frac{x^2}{(1-x)^3}\log x \;\underset{x\to 0}{\longrightarrow} 1\,, \\ h(x,y) &= y\left\{\frac{\log x}{x-y}\left[\frac{1}{4} + \frac{3}{2(1-x)} - \frac{3}{4(1-x)^2}\right] + (x \leftrightarrow y) \right. \\ &\qquad \left. - \frac{3}{4(1-x)(1-y)}\right\} \;\underset{x,y\to 0}{\longrightarrow} 0\,. \end{aligned} \tag{11.37}$$

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[1] Inami, T. and Lim, C. S., Prog. Theor. Phys. **65** (1981) 297; *ibid.* **65** (1981) 1772 (E)

We note that in the R_ξ gauge, besides the contributions of the $W^\pm$ weak boson (Fig. 11.1), in the box diagrams there are also contributions from the would-be Goldstone unphysical scalar bosons $w^\pm$ (those absorbed by $W^\pm$ to become massive). Since the coupling of these internal $w^\pm$ to the fermions $Q_i =$ u, c, t is proportional to the Q_i mass, the role of the top is dominant in these $w^\pm$ contributions. Nevertheless, the top overall part amounts to only 8% of that of the c quark. Terms proportional to $V^*_{\rm ud}V_{\rm us}$, which one expects to appear in (36), has been eliminated by the unitarity relation (31).

11.5 CP Violation

Discrete symmetries C, P, and T in particle physics have been discussed in Chap. 5. As explained there, the CPT theorem implies that T invariance in strong and electromagnetic interactions is equivalent to CP conservation in these interactions. The simplest test of the CPT theorem is the equality of the masses of a particle and its antiparticle, and the best test comes from the mass difference between K^0 and $\overline{K}^0$ which may be related to Δm :

$$\frac{m_{K^0} - m_{\overline{K}^0}}{m_{K^0}} \approx \frac{\Delta m}{m_{K^0}} \approx 9 \times 10^{-19} \, .$$

Any such difference contributes to the CP-violating parameter ϵ which we will introduce later. After the discovery in 1956 of the maximum P and C violation in weak interactions, physicists still believed that T (or CP) were nevertheless invariant in weak interactions, as in the strong and electromagnetic interactions. Imagine the great surprise when CP violations in weak decays of the neutral K system was discovered in 1964.

The standard electroweak Lagrangian with *three quark families* provides a natural framework for CP violation through the Kobayashi–Maskawa (KM) *nonzero phase* in the CKM quark mixing matrix. As remarkably shown by KM, the two left-handed doublets $(\mathrm{u}, \mathrm{d}'')$ and $(\mathrm{c}, \mathrm{s}'')$ that GIM used to cancel strangeness-changing neutral current are not enough to incorporate a complex phase in the quark flavor mixing matrix in order to have CP violation. With N quark families, the number of measurable nonzero phases is $\frac{1}{2}(N-1) \times (N-2)$ according to the discussion after (9.177). Thus N must be larger than 2 to yield a nonzero phase, i.e. at least *a third doublet* $(\mathrm{t}, \mathrm{b}'')$ is needed. It is noteworthy that this observation was made by KM in 1973 even before charm was discovered. We will call this KM theory the *standard* CP-violating mechanism.

11.5.1 General Formalism

As we have seen in the previous sections, K^0 and $\overline{K}^0$ are mixed by the weak interaction. Thus, in the presence of this type of interaction, the two states are inseparable; they form a basis for a two-dimensional subspace. Similarly, K_S and K_L form another, equivalent basis of the same space.

We may then write, for example,

$$\left|\mathrm{K}^0\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \qquad \left|\overline{\mathrm{K}}^0\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \tag{11.38}$$

Let $\psi(t)$ be an arbitrary state of such space,

$$|\psi(t)\rangle = A(t)\ \left|\mathrm{K}^0\right\rangle + B(t)\ \left|\overline{\mathrm{K}}^0\right\rangle\ =\ \begin{pmatrix} A(t) \\ B(t) \end{pmatrix} . \tag{11.39}$$

The Hamiltonian in (17) may be split into two parts; one, $H^{(0)}$, conserves strangeness (strong and electromagnetic interactions), and the other, $H^{(2)}$, violates strangeness by $|\Delta S| = 2$. Their sum $H = H^{(0)} + H^{(2)}$ should be non-Hermitian and therefore may be decomposed into a Hermitian and an anti-Hermitian part, responsible respectively for the mass and the width of unstable particles

$$\langle j|H|k\rangle = M_{jk} - \frac{\mathrm{i}}{2}\Gamma_{jk}. \tag{11.40}$$

In the two-dimensional space under consideration, H has the matrix representation

$$\begin{aligned} H &\equiv \begin{pmatrix} M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} & M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12} \\ M_{21} - \frac{\mathrm{i}}{2}\Gamma_{21} & M_{22} - \frac{\mathrm{i}}{2}\Gamma_{22} \end{pmatrix} \\ &= \begin{pmatrix} \langle \mathrm{K}^0|H^{(0)}|\mathrm{K}^0\rangle & \langle \mathrm{K}^0|H^{(2)}|\overline{\mathrm{K}}^0\rangle \\ \langle \overline{\mathrm{K}}^0|H^{(2)}|\mathrm{K}^0\rangle & \langle \overline{\mathrm{K}}^0|H^{(0)}|\overline{\mathrm{K}}^0\rangle \end{pmatrix} . \end{aligned} \tag{11.41}$$

CPT invariance of H implies that $M_{11} = M_{22} \equiv M_0$, with $M_0 = m_{\mathrm{K}^0} = m_{\overline{\mathrm{K}}^0}$, and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma_0$, with Γ_0 identified with the (common) total decay width of K^0 or $\overline{\mathrm{K}}^0$. Since by construction, both M and Γ are Hermitian, M_0 and Γ_0 are both real numbers, and $M_{21} = M_{12}^*$ and $\Gamma_{21} = \Gamma_{12}^*$. Hence

$$H \equiv \begin{pmatrix} M_0 - \frac{\mathrm{i}}{2}\Gamma_0 & M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12} \\ M_{12}^* - \frac{\mathrm{i}}{2}\Gamma_{12}^* & M_0 - \frac{\mathrm{i}}{2}\Gamma_0 \end{pmatrix} . \tag{11.42}$$

If $H^{(2)}$ is invariant under time reversal T, or equivalently under CP, one has $M_{21} = M_{12}$ and $\Gamma_{21} = \Gamma_{12}$, from which $M_{12}^* = M_{12}$ and $\Gamma_{12}^* = \Gamma_{12}$, i.e. these matrix elements are real. On the other hand, if T (or equivalently CP) is not a symmetry of $H^{(2)}$, one may either have $M_{21} \neq M_{12}$ or $\Gamma_{21} \neq \Gamma_{12}$, or both possibilities. This implies either M_{12} or Γ_{12}, or both, may be complex.

We proceed now to determine the physical eigenstates $\mathrm{K_L}$ and $\mathrm{K_S}$ having respective masses m_L and m_S and full widths Γ_L and Γ_S. Diagonalization of (42) immediately gives

$$\begin{aligned} \mathrm{K_L} &= \frac{1}{\sqrt{1+|\bar\epsilon|^2}} \left(\frac{\mathrm{K}^0 + \overline{\mathrm{K}}^0}{\sqrt{2}} + \bar\epsilon\ \frac{\mathrm{K}^0 - \overline{\mathrm{K}}^0}{\sqrt{2}} \right) \equiv \frac{\mathrm{K}_2^0 + \bar\epsilon\, \mathrm{K}_1^0}{\sqrt{1+|\bar\epsilon|^2}} , \\ \mathrm{K_S} &= \frac{1}{\sqrt{1+|\bar\epsilon|^2}} \left(\frac{\mathrm{K}^0 - \overline{\mathrm{K}}^0}{\sqrt{2}} + \bar\epsilon\ \frac{\mathrm{K}^0 + \overline{\mathrm{K}}^0}{\sqrt{2}} \right) \equiv \frac{\mathrm{K}_1^0 + \bar\epsilon\, \mathrm{K}_2^0}{\sqrt{1+|\bar\epsilon|^2}} . \end{aligned} \tag{11.43}$$

We have re-expressed K_L and K_S in terms of K_1^0 and K_2^0, the even- and odd-CP states already defined before. The parameter $\bar{\epsilon}$ is given by

$$\bar{\epsilon} = \frac{\sqrt{M_{12} - \frac{i}{2}\Gamma_{12}} - \sqrt{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}{\sqrt{M_{12} - \frac{i}{2}\Gamma_{12}} + \sqrt{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}} \equiv \frac{p-q}{p+q} \,. \tag{11.44}$$

If CP is conserved, the quantities M_{12} and Γ_{12} are real, as mentioned above, and therefore $\bar{\epsilon}$ vanishes. Then, $K_S = K_1^0$ and $K_L = K_2^0$, and there is no mixing between K_1^0 and K_2^0. Such a mixing can occur only if CP is a broken symmetry. In general, the eigenvalues of $M - i\Gamma/2$ corresponding to (43) are

$$\begin{aligned} m_L - \frac{i}{2}\Gamma_L &= M_0 - \frac{i}{2}\Gamma_0 + \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}\,, \\ m_S - \frac{i}{2}\Gamma_S &= M_0 - \frac{i}{2}\Gamma_0 - \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}\,, \end{aligned} \tag{11.45}$$

from which

$$(m_L - m_S) + \frac{i}{2}(\Gamma_S - \Gamma_L) = 2\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \equiv 2pq \,. \tag{11.46}$$

If CP is conserved, this result becomes

$$\Delta m \equiv (m_L - m_S) = 2M_{12} \;\;,\;\; \Delta\gamma \equiv (\Gamma_S - \Gamma_L) = -2\Gamma_{12} \,. \tag{11.47}$$

Let us now turn to the CP violation in neutral K mesons. In 1964, Christensen, Cronin, Fitch, and Turlay observed that K_L decays not only via the three-pion mode, $K_L \to 3\pi$, which was natural given its CP parity, but also via the two-pion mode, $K_L \to 2\pi$, which was truly unexpected. To make sure that it was the K_L and not the regenerated K_S that provoked the observed events, they surrounded the K_L beam with a great quantity of helium to eliminate any possible regeneration of K_S.

Since the same particle, K_L, can decay through two channels of opposite CP parities, it is clear that CP symmetry is violated in the pionic decay modes of K_L. This experiment is comparable in nature and significance to another result obtained eight years earlier, when it was realized that another K meson, the charged K^+, could also decay through two different channels, $K^+ \to \pi^+\pi^0$ and $K^+ \to \pi^+\pi^+\pi^-$ (or $\pi^+\pi^0\pi^0$). Now, a two-pion state in the s-wave is parity-even, while a three-pion state with zero angular momentum is parity-odd. Therefore, parity (space inversion) symmetry must be broken in these decay modes of K^+. Of course, the discovery of parity violation in weak decays was one of the most important events in modern physics (Chap. 5).

But while parity suffers maximal breakdown in weak processes (for example, in K^+ to 2π and 3π, the branching ratios are comparable in magnitudes after phase-space corrections), CP symmetry is only slightly broken (the $\mathrm{K_L}$ to 3π mode clearly dominates the 2π mode). In any case, violation of P, C, and CP symmetries by the weak interaction is a well-established fact.

CP violation manifests itself through the presence of a nonvanishing complex quantity $\bar{\epsilon}$ which may be estimated from the branching ratios

$$\rho \equiv \frac{\Gamma(\mathrm{K_L} \to \pi^+\pi^-)}{\Gamma(\mathrm{K_S} \to \pi^+\pi^-)} = \frac{\mathrm{Br}(\mathrm{K_L} \to \pi^+\pi^-)}{\mathrm{Br}(\mathrm{K_S} \to \pi^+\pi^-)} \left(\frac{\tau_\mathrm{S}}{\tau_\mathrm{L}}\right) \approx 5.1 \times 10^{-6} \,, \tag{11.48}$$

which gives $|\bar{\epsilon}| \approx \sqrt{\rho} = 2.25 \times 10^{-3}$. To obtain the complex phase of $\bar{\epsilon}$, we may proceed through (44) which takes the form

$$\bar{\epsilon} = \frac{p-q}{p+q} = \frac{p^2-q^2}{4pq+(p-q)^2} \approx \frac{p^2-q^2}{4pq} \,, \tag{11.49}$$

where the magnitude $|\bar{\epsilon}| \approx 10^{-3}$ justifies dropping the quadratic term $(p-q)^2$, which yields the approximation

$$\bar{\epsilon} \approx \mathrm{i}\, \frac{\mathrm{Im}(M_{12}) - \frac{\mathrm{i}}{2}\mathrm{Im}(\Gamma_{12})}{\Delta m + \frac{\mathrm{i}}{2}\Delta\gamma} \,. \tag{11.50}$$

Again, with $|\bar{\epsilon}| \approx 10^{-3}$, one may deduce

$$\frac{\mathrm{Im}(M_{12})}{\mathrm{Re}(M_{12})} \ll 1 \,, \ \frac{\mathrm{Im}(\Gamma_{12})}{\mathrm{Re}(\Gamma_{12})} \ll 1 \,. \tag{11.51}$$

Hence, to a very good approximation, (46) yields

$$\Delta m = 2\,\mathrm{Re}(M_{12}), \quad \Delta\gamma = -2\,\mathrm{Re}(\Gamma_{12}) \,. \tag{11.52}$$

On the other hand, as the ratio $\Delta m/\Delta\gamma = 0.477$ is experimentally known, we may write

$$\frac{\mathrm{i}}{\Delta m + \frac{\mathrm{i}}{2}\Delta\gamma} = \frac{\mathrm{e}^{\mathrm{i}(43.37)^\circ}}{\sqrt{2.098}\ \Delta m} \approx \frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{2}\ \Delta m} \,, \tag{11.53}$$

and the approximate expression for $\bar{\epsilon}$, (50) becomes

$$\bar{\epsilon} \approx \frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{2}\ \Delta m} \left[\mathrm{Im}(M_{12}) - \frac{\mathrm{i}}{2}\mathrm{Im}(\Gamma_{12})\right] . \tag{11.54}$$

In order to determine the phase of $\bar{\epsilon}$, or equivalently its real part, we consider again the charge asymmetry $\delta(t)$ as defined in (14). This quantity was previously obtained assuming exact CP conservation, i.e. with $\mathrm{K_L} = \mathrm{K}_2^0$ and

$K_S = K_1^0$. Now, in the presence of CP violation, one should replace K_L and K_S by their exact expressions (43). This replacement transforms (14) into

$$\delta(t) = 2\left[\mathrm{Re}(\bar\epsilon) + \mathrm{e}^{-(\Gamma_S+\Gamma_L)\frac{t}{2}}\ \cos(\Delta m\, t)\right], \tag{11.55}$$

which includes the CP violation effect. This result tells us that at $t \approx \tau_S$ the time oscillation yields a measure of Δm, while at $t \gg \tau_S$ the amplitude of $\delta(t)$ directly yields $\mathrm{Re}(\bar\epsilon)$. Figure 11.3 clearly shows the nonvanishing value of $\mathrm{Re}(\bar\epsilon)$. We may understand the above result as follows: at large t only K_L survives; because of CP violation as shown in (43), K^0 and $\overline{K}^0$ exist in unequal proportions in K_L, and their difference is given by $2\bar\epsilon$.

11.5.2 Model-Independent Analysis of $K_L \to 2\pi$

Let us now examine in some detail the CP-violating modes $K_L \to \pi^+ + \pi^-$ and $K_L \to \pi^0 + \pi^0$. The eigenstates (43) indicate that there are two possible not mutually exclusive scenarios in which these decays may proceed.

First, it is the K_2^0 component of K_L itself that decays into two pions. As K_2^0 and $\pi\pi$ states have opposite CP parities, this decay mode can occur only if the transition violates CP symmetry. As this is a direct violation by the amplitude itself, the effective coupling strength for this interaction is of the order of 10^{-3} of the strength of the CP-conserving interaction. It is a $|\Delta S| = 1$ transition and is referred to as a *milliweak* transition. In $K_L \to \pi + \pi$, its effects are parameterized by a quantity called ϵ' defined later.

In the second scenario, the decay $K_L \to \pi + \pi$ is viewed as being due to the $\bar\epsilon K_1^0$ component, with $\bar\epsilon \neq 0$. In contrast to the first scenario, in this case the decay amplitude, which is that of $K_1^0 \to \pi\pi$ multiplied by $\bar\epsilon$, exactly conserves CP. The CP violation actually observed in the K_L decay stems from the K^0–$\overline{K}^0$ mixing through the mass matrix in (41). It is this mass matrix that violates CP. As the violation occurs through $|\Delta S| = 2$ transitions, which are $\sim G_F^2$, Wolfenstein dubbed it *superweak*.

Of course, it is quite possible that the CP violation in $K_L \to \pi + \pi$ proceeds via both scenarios. Actually, it is what happens in the *standard* CP-violating mechanism defined earlier. The two scenarios are represented by the diagrams in Fig. 11.4.

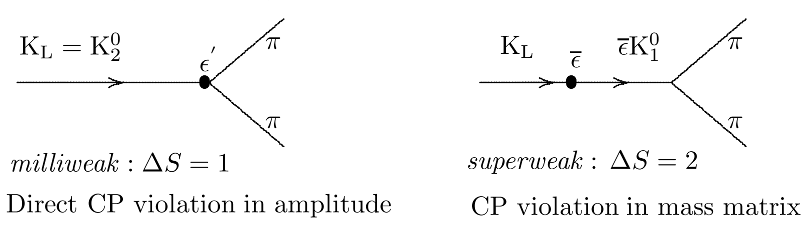

Fig. 11.4. Two scenarios of CP violation in $K_L \to 2\pi$

In order to show how experiment may bring out these two scenarios, let us introduce the following complex quantities which give the ratios of the two-pion decay amplitudes of $\mathrm{K_L}$ and $\mathrm{K_S}$:

$$\begin{aligned} \eta^{+-} \equiv |\eta^{+-}|e^{\mathrm{i}\phi^{+-}} &\equiv \frac{A(\mathrm{K_L} \to \pi^+\pi^-)}{A(\mathrm{K_S} \to \pi^+\pi^-)} \; , \\ \eta^{00} \equiv |\eta^{00}|e^{\mathrm{i}\phi^{00}} &\equiv \frac{A(\mathrm{K_L} \to \pi^0\pi^0)}{A(\mathrm{K_S} \to \pi^0\pi^0)} \; . \end{aligned} \tag{11.56}$$

We will discuss how their complex phases and their magnitudes can be determined. In particular, a very precise measure of their magnitudes will teach us a great deal about the nature and the mechanism of the CP violation.

A method used in such experiments is based on the interference phenomenon, analogous to the one considered in Sect. 11.2. At time $t = 0$, a beam of pure K^0 mesons is produced. Using (43), its amplitude of transition into a 2π final state is given by

$$A(K^0 \to 2\pi) = \sqrt{\frac{1+|\bar{\epsilon}|^2}{2|1+\bar{\epsilon}|^2}}\,[A(\mathrm{K_S} \to 2\pi) + A(\mathrm{K_L} \to 2\pi)] \; . \tag{11.57}$$

The time evolution of $\mathrm{K_L}$ and $\mathrm{K_S}$ are

$$\mathrm{K_{S,L}}(t) = \mathrm{K_{S,L}}(0) \exp\left[\frac{-t\,\Gamma_{\mathrm{S,L}}}{2}\right] \; \mathrm{e}^{-\mathrm{i}(m_{\mathrm{S,L}})t} \; . \tag{11.58}$$

From (56), the t-dependence of the probability for the $\pi^+\pi^-$ or $2\pi^0$ mode is

$$I_{\pi\pi}(t) = I_{\pi\pi}(0)\Big[\mathrm{e}^{-\Gamma_S t} + |\eta|^2 \mathrm{e}^{-\Gamma_L t} + 2|\eta|\mathrm{e}^{-(\Gamma_S+\Gamma_L)\frac{t}{2}}\cos(\Delta m\, t - \phi)\Big] \; . \tag{11.59}$$

As Δm is already known from independent measurements (Sect. 11.2), observation of the oscillations of $I_{\pi\pi}(t)$ for t of the order of a few τ_{S} yields the phase ϕ, while for $t \gg \tau_{\mathrm{S}}$, when the first term in (59) has died down, one can make a measure of $|\eta|$. Data from the two most recent experiments give

$$\begin{array}{rll} \phi^{+-} & = (46.9 \pm 2.2)^\circ & \text{CERN}\; , \\ \phi^{+-} & = (43.53 \pm 0.97)^\circ & \text{FNAL}\; , \\ \phi^{00} & = (47.1 \pm 2.8)^\circ & \text{CERN}\; , \\ \Delta\phi \equiv \phi^{00} - \phi^{+-} & = (0.62 \pm 1.03)^\circ & \text{FNAL}\; , \\ |\frac{\eta^{00}}{\eta^{+-}}| & = (0.9931 \pm 0.002 & \text{CERN}\; , \\ |\frac{\eta^{00}}{\eta^{+-}}| & = (0.9904 \pm 0.0084 \pm 0.0036) & \text{FNAL}\; . \end{array}$$

In order to show how these results serve to distinguish the two CP-violating scenarios, we have to make a detailed analysis of the decay amplitudes. The final products of K^0 decays, $\pi^+\pi^-$ and $2\pi^0$, have only zero orbital angular momentum (s-wave). These two pions cannot have isospin 1 (forbidden by Bose statistics) but may have isospin 0 or 2. The isospin decomposition of the decay amplitudes may be written down with the help of the Clebsch–Gordan coefficients as follows:

$$A(\mathrm{K}^0 \to \pi^+ + \pi^-) = \frac{1}{\sqrt{3}}[A_2 + \sqrt{2}A_0] \; ,$$
$$A(\mathrm{K}^0 \to \pi^0 + \pi^0) = \frac{1}{\sqrt{3}}[\sqrt{2}A_2 - A_0] \; , \tag{11.60}$$

where A_0 and A_2 are respectively weak decay amplitudes into isospin $I = 0$ and isospin $I = 2$ states of the two-pion system:

$$A_0 \equiv \langle \pi\pi, I = 0|H_\mathrm{w}|\mathrm{K}^0\rangle \quad , \quad A_2 \equiv \langle \pi\pi, I = 2|H_\mathrm{w}|\mathrm{K}^0\rangle \; . \tag{11.61}$$

As K^0 has isospin $1/2$, the matrix element A_2 is nonvanishing only if H_w behaves as an isospin $I = 3/2$ or $I = 5/2$ operator. Similarly, for a nontrivial A_0, H_w must transform as an isospin $I = 1/2$ operator. Now, from the empirical rule of isospin weak transitions $\Delta I = 1/2$ (Chap. 6), one should expect $a_{1/2} = \left\langle \beta \left| H_\mathrm{w}^{I=1/2} \right| \alpha \right\rangle$ to be substantially larger than $a_{3/2} = \left\langle \beta \left| H_\mathrm{w}^{I=3/2} \right| \alpha \right\rangle$. In fact, it is found in various hadronic decays of strange particles α into nonstrange particles β that their ratio, $|a_{1/2}/a_{3/2}|$, varies between 15 and 30. It follows that $|A_0| \gg |A_2|$. From $\mathrm{K_S} \to \pi^+\pi^-$ and $\mathrm{K_S} \to \pi^0\pi^0$ experimental data, one gets $|A_0/A_2| \approx 22$.

From our previous discussion, it is expected that in the case of exact CP symmetry, A_0 and A_2 may be taken to be real, or more exactly their relative phase may be chosen to be zero. But if CP symmetry is broken and if this symmetry breakdown resides in the decay amplitude, as the case of the first scenario, the isospin amplitudes A_I are in general complex. This implies necessarily complex matrix elements Γ_{12}, i.e. $\mathrm{Im}\,\Gamma_{12} \neq 0$.

Now for the $\overline{\mathrm{K}}^0$ case, the corresponding two-pion $\overline{\mathrm{K}}^0$ decay amplitudes can be obtained from (60) by assuming exact CPT symmetry. Let us define

$$\bar{A}_0 \equiv \langle \pi\pi, I = 0|H_\mathrm{w}|\overline{\mathrm{K}}^0\rangle \quad , \quad \bar{A}_2 \equiv \langle \pi\pi, I = 2|H_\mathrm{w}|\overline{\mathrm{K}}^0\rangle \; . \tag{11.62}$$

Upon CPT transformations, $|\mathrm{K}^0\rangle \mapsto -\langle\overline{\mathrm{K}}^0|$, $\langle \pi\pi, I = 0| \mapsto |\pi\pi, I = 0\rangle$, and $\langle \pi\pi, I = 2| \mapsto |\pi\pi, I = 2\rangle$, the assumed CPT symmetry of H_w yields

$$\bar{A}_0 = -A_0^* \quad , \quad \bar{A}_2 = -A_2^* \; ; \tag{11.63}$$

then from (60), the $\overline{\mathrm{K}}^0$ decay amplitudes are

$$A(\overline{\mathrm{K}}^0 \to \pi^+\pi^-) = -\frac{1}{\sqrt{3}}[A_2^* + \sqrt{2}A_0^*] \; ,$$
$$A(\overline{\mathrm{K}}^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}}[-\sqrt{2}A_2^* + A_0^*] \; . \tag{11.64}$$

It remains to include the effects of the final-state *strong interaction* that acts between the π mesons in the final states, i.e. the decay products of the K_L. These effects are parameterized, as usual, by the pion–pion scattering phase-shifts $\delta_0(m_K)$ and $\delta_2(m_K)$ for isospin $I = 0$ and $I = 2$ respectively. These pion–pion phase-shifts are measured and known. With these strong interaction effects included, the four relevant amplitudes are

$$\begin{aligned}
\sqrt{3}\,A(\mathrm{K}^0 \to \pi^+\pi^-) &= \left[e^{\mathrm{i}\delta_2} A_2 + \sqrt{2}\, e^{\mathrm{i}\delta_0} A_0\right] \ , \\
\sqrt{3}\,A(\mathrm{K}^0 \to \pi^0\pi^0) &= \left[\sqrt{2}\, e^{\mathrm{i}\delta_2} A_2 - e^{\mathrm{i}\delta_0} A_0\right] \ , \\
\sqrt{3}\,A(\overline{\mathrm{K}}^0 \to \pi^+\pi^-) &= -\left[e^{\mathrm{i}\delta_2} A_2^* + \sqrt{2}\, e^{\mathrm{i}\delta_0} A_0^*\right] \ , \\
\sqrt{3}\,A(\overline{\mathrm{K}}^0 \to \pi^0\pi^0) &= \left[-\sqrt{2}\, e^{\mathrm{i}\delta_2} A_2^* + e^{\mathrm{i}\delta_0} A_0^*\right] \ .
\end{aligned} \tag{11.65}$$

Re-expressing K^0 and $\overline{\mathrm{K}}^0$ in terms of K_L and K_S, one gets

$$\eta^{+-} = \epsilon + \epsilon' \ , \ \ \eta^{00} = \epsilon - 2\epsilon' \ , \ \ \text{with} \tag{11.66}$$

$$\epsilon = \bar{\epsilon} + \mathrm{i}\,\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \, , \ \ \text{and} \tag{11.67}$$

$$\epsilon' = \frac{\mathrm{i}\, w}{\sqrt{2}} e^{\mathrm{i}(\delta_2 - \delta_0)} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right] , \tag{11.68}$$

where $w = \mathrm{Re}\, A_2/\mathrm{Re}\, A_0 \approx |A_2/A_0| \approx 1/22$.

From the above results we learn the following. First, observation of CP violation effects generally involves two interfering transition amplitudes; for ϵ', it is the interference between A_0 and A_2, while for ϵ, it is the interference between the amplitudes $\mathrm{K}^0 \to 2\pi$ and $\overline{\mathrm{K}}^0 \to 2\pi$ through $\bar{\epsilon}$. Second, the parameter ϵ' stems exclusively from the first scenario through nonzero values of $\mathrm{Im}\, A_0$ or $\mathrm{Im}\, A_2$, or of both of them. With (68), the phase of ϵ' as taken from $(\mathrm{i}\ \mathrm{e}^{\mathrm{i}(\delta_2-\delta_0)})$ is $\frac{\pi}{2} + \delta_2 - \delta_0$. Since both δ_2 and δ_0 are experimentally known, the phase of ϵ' is $48° \pm 4°$. In fact, the direct CP violation through the first scenario is completely determined from the knowledge of $\eta^{+-} - \eta^{00} = 3\epsilon'$ and $|\eta^{00}/\eta^{+-}|^2 = 1 - 6\,\mathrm{Re}(\epsilon'/\epsilon)$. Due to an unfortunate circumstance, ϵ' is strongly suppressed by w, reduced by the $\Delta I = 1/2$ rule (Chap. 6), and also by a possibly small difference $(\mathrm{Im}A_2/\mathrm{Re}A_2) - (\mathrm{Im}A_0/\mathrm{Re}A_0)$. Note that while $A_2 \ll A_0$, the ratios of their imaginary/real parts may be comparable, since $\mathrm{Re}A_2$ is in the denominator. From available data on η^{+-} and η^{00}, one has

$$|\epsilon'/\epsilon| = \begin{array}{ll} 0.00074 \pm 0.00059 & \text{FNAL, E731}\ , \\ 0.0023 \pm 0.00065 & \text{CERN, NA31}\ . \end{array} \tag{11.69}$$

The question of whether ϵ' vanishes or not is still inconclusively answered.

Finally, from the $\Delta I = 1/2$ rule, the isospin scalar amplitude predominates in K$\to 2\pi$, hence the CP-violating component in Γ_{12}, i.e. $\mathrm{Im}\,\Gamma_{12}$ must come mainly from $\mathrm{Im}\,A_0$, so that

$$\mathrm{Im}(\Gamma_{12}) \approx \Gamma_{\mathrm{S}}\ \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \approx 2\Delta m\ \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\ . \tag{11.70}$$

Substituting this result for $\mathrm{Im}\,\Gamma_{12}$ in the expression (54) for $\bar{\epsilon}$, and using (67), one gets

$$\begin{aligned}\epsilon = \bar{\epsilon} + \mathrm{i}\,\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} &\approx \frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{2}}\Big[\frac{\mathrm{Im}(M_{12})}{\Delta m} - \mathrm{i}\,\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\Big] + \mathrm{i}\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\\ &\approx \frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{2}}\Big[\frac{\mathrm{Im}(M_{12})}{2\,\mathrm{Re}(M_{12})} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\Big]\ .\end{aligned} \tag{11.71}$$

The expressions for ϵ' and ϵ given in (68) and (71) have the following important property. As a general rule in physics, the absolute phase of an isolated amplitude has no physical meaning; it is only when it is measured relatively to the phase of another amplitude that it takes on a meaning. Therefore, to see whether there are actually CP violation effects or not, it is necessary to determine the relative phase of two interfering amplitudes in some appropriate weak transition. Also the absolute phase of a state function has no physical meaning, thus one expects that making the changes

$$\mathrm{K}^0 \to \mathrm{e}^{\mathrm{i}\alpha}\mathrm{K}^0, \qquad \overline{\mathrm{K}}^0 \to \mathrm{e}^{-\mathrm{i}\alpha}\overline{\mathrm{K}}^0 \qquad \text{or equivalently,}$$

$$\frac{\mathrm{Im}(A_{0,2})}{\mathrm{Re}(A_{0,2})} \to \frac{\mathrm{Im}(A_{0,2})}{\mathrm{Re}(A_{0,2})} + \alpha \quad , \quad \frac{\mathrm{Im}(M_{12})}{\mathrm{Re}(M_{12})} \to \frac{\mathrm{Im}(M_{12})}{\mathrm{Re}(M_{12})} - 2\alpha\ , \tag{11.72}$$

would leave ϵ and ϵ' unchanged. This is indeed the case, which shows that these parameters are physically meaningful and truly represent the CP violation effects in K decays.

The phenomenological model-independent analysis given above is served to confront experimental data with different CP-violating mechanisms provided by theoretical models. These theories must give predictions for $\mathrm{Im}(M_{12})$ and $\mathrm{Im}(\Gamma_{12})$ (or Im $(A_{0,2})$) from which we obtain ϵ and ϵ'.

In the standard model we will show that the gluonic penguin diagram implies $\mathrm{Im}\,A_0 \neq 0$, i.e. direct CP violation in the decay amplitude (first scenario). Thus a measure of ϵ' through that of η^{+-}/η^{00} is crucial for the validity of the standard KM mechanism of CP violation, which predicts a small but nonvanishing value of ϵ'. An exactly vanishing value of ϵ' would be a fatal blow to this *standard* KM mechanism.

In the second scenario (superweak) discussed below, CP violation in $\mathrm{K_L} \to 2\pi$ is only due to an unknown mechanism which has, by assumption, an effective $\Delta S = 2$ *complex* mass matrix element M_{12}. This complex M_{12}, put in by hand to fit $K_L \to 2\pi$ data, is translated into an effective $\bar{\epsilon} \neq 0$. In superweak model, except in the neutral K meson system, the discrete symmetry CP is nowhere violated. This scenario is completely different from the standard KM nonzero phase of the electroweak theory which predicts that, outside the K meson system with nonzero ϵ', large CP violations are expected in many decay channels of B mesons.

11.5.3 The Superweak Scenario

As already mentioned, the decay amplitudes in the second scenario are CP conserving. It is only because of $\bar{\epsilon} \neq 0$ that $K_L \to 2\pi$. In this case, the amplitudes A_0 and A_2 can be taken as real, so ϵ' is *identically zero*. The CP violation observed in K_L decay would arise from the K_1^0–K_2^0 mixing through a complex mass matrix element M_{12}. Note that Γ_{12} is, in contrast, a real number.

The K_1^0–K_2^0 mixing may be calculated as

$$\begin{aligned}\langle K_1^0|H_w|K_2^0\rangle &= \left(\frac{1}{\sqrt{2}}\right)^2 \langle K^0 - \overline{K}^0|H_w|K^0 + \overline{K}^0\rangle \\ &= \tfrac{1}{2}\left[\langle K^0|H_w|\overline{K}^0\rangle - \langle \overline{K}^0|H_w|K^0\rangle\right] \\ &= \tfrac{1}{2}\left(M_{12} - M_{12}^*\right) = \mathrm{i}\,\mathrm{Im}(M_{12})\ . \end{aligned} \tag{11.73}$$

Since $\mathrm{Im}\,A_0 = 0$ and $\mathrm{Im}\,\Gamma_{12} = 0$, this yields, through (54) and (67),

$$|\epsilon| = |\bar{\epsilon}| = \frac{\mathrm{Im}(M_{12})}{\sqrt{2}\Delta m} \quad , \quad \epsilon' = 0\,. \tag{11.74}$$

Since Δm is of the order of G_F^2, the superweak mixing matrix Im (M_{12}) is further reduced below this level by ϵ. The effective coupling strength of the superweak, which is proportional to Im (M_{12}), has roughly the magnitude of $|\epsilon| G_F m_c^2/6\pi^2 \approx 10^{-10}$ compared to weak interaction strength $\sim G_F$. Nevertheless, the superweak interaction may still manifest itself through the mixing factor $\bar{\epsilon}$ because in (74), the denominator Δm is also very small.

In this second scenario, all CP violation effects depend on the single parameter $\bar{\epsilon}$. It predicts

$$\eta^{+-} = \frac{\langle \pi^+\pi^-|K_L\rangle}{\langle \pi^+\pi^-|K_S\rangle} = \frac{\langle \pi^+\pi^-|\bar{\epsilon}K_1^0\rangle}{\langle \pi^+\pi^-|K_1^0\rangle} = \bar{\epsilon}\ , \tag{11.75}$$

$$\eta^{00} = \frac{\langle \pi^0\pi^0|K_L\rangle}{\langle \pi^0\pi^0|K_S\rangle} = \frac{\langle \pi^0\pi^0|\bar{\epsilon}K_1^0\rangle}{\langle \pi^0\pi^0|K_1^0\rangle} = \bar{\epsilon}\ , \tag{11.76}$$

$$\text{so}\ \ \eta^{+-} = \eta^{00}\ \ \text{and}\ \ \phi^{+-} = \phi^{00} = \phi^{\bar{\epsilon}} = \arctan\frac{2\Delta m}{\Delta\gamma} = (43.37 \pm 0.2)^\circ. \tag{11.77}$$

To date, all data on $\eta^{+-}|$, $|\eta^{00}|$, ϕ^{+-}, and ϕ^{00} are consistent with the superweak scenario. This mechanism also makes the prediction that $K_L \to 2\,\pi$

and $\mathrm{K_S} \to 3\,\pi$ give rise to quantitatively equal CP violation effects:

$$\eta^{+-0} \equiv \frac{\langle \pi^+\pi^-\pi^0|\mathrm{K_S}\rangle}{\langle \pi^+\pi^-\pi^0|\mathrm{K_L}\rangle} = \bar{\epsilon} = \eta^{000} \equiv \frac{\langle \pi^0\pi^0\pi^0|\mathrm{K_S}\rangle}{\langle \pi^0\pi^0\pi^0|\mathrm{K_L}\rangle}\,. \tag{11.78}$$

So $\eta^{+-} = \eta^{00} = \eta^{+-0} = \eta^{000}$. Unfortunately, the three-pion $\mathrm{K_S}$ decay modes are extremely difficult to detect because they are suppressed by the smallness of the CP symmetry breakdown (10^{-3}) and by the reduction of the available phase space in the final state.

As shown in the next subsection, the standard model also gives results for ϵ close to the data, although, as already noted, CP violation operates through both scenarios and $\epsilon' \neq 0$ is predicted.

11.5.4 Calculations of ϵ and ϵ' in the Standard Model

In order to obtain ϵ and ϵ', defined in (68) and (71), we compute the quantities M_{12}, A_0 and A_2. Let us first introduce the Wolfenstein parameterization of the CKM matrix (V_{CKM}) which is particularly suitable for CP violation analyses.

As in any version, V_{CKM} possesses four parameters: three Euler angles and one phase. Wolfenstein's version expands the matrix elements in power of $\lambda = \sin\theta_{\mathrm{C}} = 0.2205 \pm 0.0018$ (θ_{C} is the Cabibbo angle). The other three parameters are A, ρ, and η. To order λ^3 for the real parts and order λ^4 for the imaginary parts, V_{CKM} reads

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3[\rho - \mathrm{i}\eta(1-\frac{\lambda^2}{2})] \\ -\lambda & 1-\frac{\lambda^2}{2} - \mathrm{i}\eta A^2\lambda^4 & A\lambda^2(1+\mathrm{i}\eta\lambda^2) \\ A\lambda^3(1-\rho-\mathrm{i}\eta) & -A\lambda^2 & 1 \end{pmatrix}\,. \tag{11.79}$$

The parameter η represents the complex phase responsible for CP violation. A, ρ, and η can be extracted from data on B meson decays (see Chap. 16), with the results: $A = 0.794 \pm 0.054$, $\sqrt{\rho^2+\eta^2} = 0.363 \pm 0.073$.

Calculation of ϵ. Contributions to the matrix element M_{12} in the standard model are represented by diagrams in Fig. 11.1. As expressed by (52), its real part, $\Delta m/2$, has been calculated in (36), with the final result obtained by taking the real part of a certain complex expression. Up to a factor of 2, due to the uncertainty of the parameter B in (35), the imaginary part is

$$\begin{aligned}\mathrm{Im}(M_{12}) = &\frac{G_{\mathrm{F}}^2}{12\pi^2} f_{\mathrm{K}}^2 m_{\mathrm{K}} m_{\mathrm{c}}^2 \Big\{ g(x_{\mathrm{c}})\ \mathrm{Im}(V_{\mathrm{cd}}^* V_{\mathrm{cs}})^2 + \frac{x_t}{x_{\mathrm{c}}}\ g(x_{\mathrm{t}})\ \mathrm{Im}(V_{\mathrm{td}}^* V_{\mathrm{ts}})^2 \\ &+ 2h(x_{\mathrm{c}}, x_{\mathrm{t}})\ \mathrm{Im}(V_{\mathrm{td}}^* V_{\mathrm{ts}} V_{\mathrm{cd}}^* V_{\mathrm{cs}}) \Big\} B\,,\end{aligned} \tag{11.80}$$

where $g(x)$ and $h(x,y)$ are known from (37). Therefore, the calculation reduces to that of the product of four complex matrix elements $V_{\mathrm{qd}}^* V_{\mathrm{qs}} V_{\mathrm{q'd}}^* V_{\mathrm{q's}}$.

Let us start by studying some properties of the CKM matrix related to CP violation. Define the product

$$\Delta_{\gamma k} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i} \,, \tag{11.81}$$

where $(\alpha, \beta, \gamma) = (1, 2, 3) \equiv$ (u,c,t) or any other cyclic permutation, and similarly, $(i, j, k) = (1, 2, 3) \equiv$ (d, s, b). These nine complex numbers $\Delta_{\gamma k}$, though very different in magnitudes and in their real parts, have exactly equal imaginary parts. Their common imaginary parts will be denoted by J, from Cecilia Jarlskog who was the first to point out this remarkable property. As will be shown further on,

$$\mathrm{Im}(\Delta_{\gamma k}) \equiv \mathrm{Im}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}) = J \sum_{\gamma,k} \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} \,. \tag{11.82}$$

The term J is a universal number in the sense that it does not depend on how the CKM matrix is parameterized. It shares this property with $|V_{\alpha i}|$. In addition, J as well as $|V_{\alpha i}|$ are also invariant to the phase redefinition of the quark fields that define the matrix representation, e.g. $q \to q\mathrm{e}^{\mathrm{i}\theta}$ implies $V_{\gamma k} \to V_{\gamma k} \mathrm{e}^{\mathrm{i}(\theta_k - \theta_\gamma)}$. J is both invariant to phase redefinition of the quark fields (called rephasing-invariant) and independent on the parameterization of the CKM matrix. Since any physical quantity that violates CP symmetry is proportional to J, that quantity must equally possess these properties.

The reason all nine $\Delta_{\gamma k}$ have the same imaginary part can be seen as follows. First multiply both sides of the unitarity relation

$$V_{\alpha i} V^*_{\alpha j} = -\sum_{\delta \neq \alpha} V_{\delta i} V^*_{\delta j} \,, \qquad \text{with } i \neq j \,, \tag{11.83}$$

by $V_{\beta j} V^*_{\beta i}$, and using the cyclic character of the indices, the $\Delta_{\gamma k}$ in (81) is transformed into

$$\Delta_{\gamma k} = -\Delta^*_{\alpha k} - |V_{\beta i}|^2 |V_{\beta j}|^2 \,. \tag{11.84}$$

From (84) it is evident that the nine $\Delta_{\gamma k}$ have equal imaginary parts, and that there exists just one independent $\Delta_{\gamma k}$, the other eight being expressible in terms of it and of the magnitudes of the CKM matrix elements. The common imaginary part is given by

$$J = A^2 \lambda^6 \eta \,, \ \text{ or } \ J = |(c_{13})^2\, c_{23}\, c_{12}\, s_{12}\, s_{13}\, s_{23}\, \sin\delta_{13}| \tag{11.85}$$

in the Wolfenstein's version or in the version (9.178) of V_{CKM}.

It can also be shown that J is given by twice the area of any one of the six triangles defined by the following six unitarity relations: three obtained by fixing any two columns i and j, and three others obtained by fixing any two rows β and γ :

$$\sum_{\substack{\alpha=1 \\ i\neq j}}^{3} V_{\alpha i} V^*_{\alpha j} = 0 \quad , \quad \sum_{\substack{k=1 \\ \beta\neq\gamma}}^{3} V_{\beta k} V^*_{\gamma k} = 0.$$

These six relations may be represented by six triangles in the complex plane. For example, the three complex numbers considered as vectors $\mathbf{A}_1 = V_{11}V_{13}^*$, $\mathbf{A}_2 = V_{21}V_{23}^*$, and $\mathbf{A}_3 = V_{31}V_{33}^*$, which sum up to zero, define one such triangle. They are called *unitarity triangles*, their significance to heavy flavor physics will be discussed later in Chap. 16.

Although very dissimilar in their shapes, all those triangles have equal areas, given by $\frac{1}{2}|\mathbf{A}_1||\mathbf{A}_2|\sin(\mathbf{A}_1\cdot\mathbf{A}_2)$, which is $\frac{1}{2}\,\mathrm{Im}[\mathbf{A}_1\cdot\mathbf{A}_2^*] = \frac{1}{2}\,\mathrm{Im}\,\Delta_{32} = \frac{1}{2}\,J$. Just as the area of a triangle is given by the lengths of its sides, so J is also given by the various $|V_{ij}|$. In particular J vanishes if one of the nine V_{ij} does. Hence the necessary condition for CP violation (i.e. $J \neq 0$) is that none of the nine matrix elements V_{ij} is zero. Returning now to (80), we express the three factors found in it as

$$\begin{aligned}
\mathrm{Im}(V_{\rm cd}^* V_{\rm cs})^2 &= -2J\ , \\
\mathrm{Im}(V_{\rm td}^* V_{\rm ts})^2 &= 2A^2\lambda^4(1-\rho)J\ , \\
\mathrm{Im}(V_{\rm td}^* V_{\rm ts} V_{\rm cd}^* V_{\rm cs}) &= +J\ ,
\end{aligned} \tag{11.86}$$

where $J = A^2\lambda^6\eta$. From (54), (71), (80) and the above equation, one gets

$$|\epsilon| = \frac{G_{\rm F}^2 f_{\rm K}^2 m_{\rm c}^2\, m_{\rm K}}{6\sqrt{2}\pi^2\Delta m}\ J\ \left[-g(x_{\rm c}) + A^2\,\lambda^4\,(1-\rho)\,\frac{x_{\rm t}}{x_{\rm c}} g(x_{\rm t}) + h(x_{\rm c}, x_{\rm t})\right] B\ . \tag{11.87}$$

We have approximated $|\epsilon| \approx |\bar\epsilon|$ by neglecting $\mathrm{Im}(A_0)$, i.e. by neglecting ϵ' in ϵ, [note that $|\epsilon'/\epsilon| = \mathcal{O}(10^{-4})$]. Since J is small, it is not surprising that ϵ is in agreement with experiment. With the ϵ measurement alone, the standard KM mechanism cannot be differentiated from the superweak scenario, hence the crucial role of ϵ' for testing different CP-violating mechanisms.

How do we compute ϵ' ? Whereas ϵ is related to the $\Delta S = 2$ mixing matrix M_{12} as depicted in Fig. 11.1, the parameter ϵ' describes direct CP violation with the $\Delta S = 1$ transition s→ d shown in Fig. 11.5a. As will be shown later, for large W boson mass, this diagram is equivalent to the 'penguin' diagram of Fig. 11.5b.

The calculation of ϵ' is more subtle because it involves the amplitudes A_0 and A_2, the first of which being dominant, according to the $\Delta I = 1/2$ empirical rule. It will be seen later (Chap. 16) that QCD does indeed amplify the $I = 1/2$ transitions at the expense of the $I = 3/2$ transitions, giving a qualitative explanation for the $\Delta I = 1/2$ rule. There are several reasons for the penguin diagram to be the key to the calculation of A_0, and hence to direct CP violation in the amplitude (first scenario).

First, the transition s→d changes strangeness by one unit, exactly as required by the first scenario.

Second, we will see later in (94) that the gluonic penguin operator has the form $\alpha_s[\bar d\gamma_\mu(1-\gamma_5)\lambda^j s]\,[\bar q\gamma^\mu\lambda^j q]$. This is an $I = 1/2$ operator, because it is the product of an $I = 1/2$ operator, $\bar d\gamma_\mu(1-\gamma_5)\lambda^j s$, with an $I = 0$

gluonic current $\bar{q}\gamma^{\mu}\lambda^{j}q$. Thus the matrix element A_0 of an $I = 1/2$ transition is obtained. Let us mention in passing that A_2 may be obtained from a diagram similar to that in Fig. 11.5, with the gluon replaced by a photon or a Z^0 and is called electroweak penguin. Now there is an electromagnetic or a weak neutral current of isospins 0 and 1 coupled to the $I = 1/2$ operator to give, among others, isospin-$3/2$ transition and hence A_2.

Finally, the penguin diagram involves the matrix elements $V^{*}_{\mathrm{Qd}}V_{\mathrm{Qs}}$ for Q= u, c, t; hence the A_0 depends in particular on $V^{*}_{\mathrm{cd}}V_{\mathrm{cs}}$ and $V^{*}_{\mathrm{td}}V_{\mathrm{ts}}$, and so must be complex. These complex matrix elements give rise to direct CP violation in the amplitude.

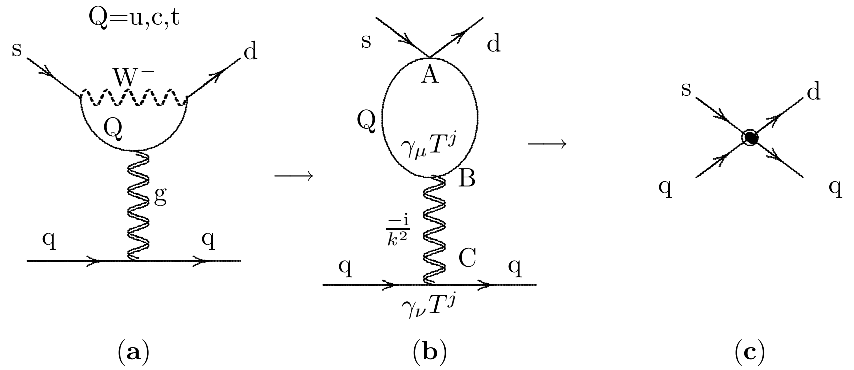

Fig. 11.5. (**a**) $\mathrm{s} + \mathrm{q} \to \mathrm{d} + \mathrm{q}$ transition; (**b**) penguin: same as (**a**) but the W propagator is squeezed into the point A; (**c**) $H^{\mathrm{glu}}_{\mathrm{pen}}$ [see (93)] is a local operator because the gluon propagator $1/k^2$ is compensated by k^2 of the loop integral

11.5.5 The Gluonic Penguin and $|\epsilon'/\epsilon|$

To lowest order of the Fermi coupling G_{F}, the flavor-changing neutral current $\mathrm{s} \to \mathrm{d}$ is forbidden by the GIM cancelation mechanism (Chap. 9). Induced by QCD, the $\mathrm{s} \to \mathrm{d}$ transition with one gluon emitted, as shown by Fig. 11.5a, gives rise to an effective interaction $H^{\mathrm{glu}}_{\mathrm{pen}}$ which yields CP violation in the decay amplitude $\mathrm{K_L} \to 2\pi$ with a strength $\sim G_{\mathrm{F}}\alpha_{\mathrm{s}}/\pi$.

Our purpose is to compute this effective interaction $H^{\mathrm{glu}}_{\mathrm{pen}}$. As can be shown below, the diagram 11.5a gives the same result as the diagram 11.5b, once the sum over Q=u, c, t is performed to overcome the divergences in the loop integrals of the diagrams 11.5a, b.

We first compute the Q quark loop represented by Fig. 11.5b. At the gluon vertex (shown by B in Fig. 11.5b), there is a QCD quark current $\bar{Q}\gamma_{\mu}T^{j}Q$ which contains the SU(3) color matrix $T^{j} \equiv \frac{1}{2}\lambda^{j}$. On the other hand, the weak interaction vertex (shown by A in Fig. 11.5b) is a four-point vertex of the type $[\bar{d}\gamma_{\mu}(1-\gamma_5)Q]\,[\bar{Q}\gamma^{\mu}(1-\gamma_5)s]$ which of course is a color singlet. To calculate the effective interaction operator which is under consideration, a trace over the color labels is to be taken in the quark loop integral.

Since the expression of the transition amplitude contains an explicit color matrix T^j from the gluon vertex but none from the weak interaction four-point vertex, it is convenient to make a decomposition of the color content of the latter. This can be done with the help of the identity (Problem 11.5)

$$\delta_{eh}\delta_{gf} = \frac{1}{3}\delta_{ef}\delta_{gh} + \frac{1}{2}\sum_{j=1}^{8}(\lambda^j)_{ef}(\lambda^j)_{gh} \ , \tag{11.88}$$

where e, f, g, h are color indices running from 1 to 3. Actually, this relation is very useful in QCD corrections to weak decays, and we will exploit it again in Chap. 16. Using (88) together with the Fierz rearrangement, we have

$$\begin{aligned}[\bar{d}\gamma_\mu(1-\gamma_5)\,Q][\bar{Q}\gamma^\mu(1-\gamma_5)s] &= \frac{1}{3}[\bar{d}\gamma_\mu(1-\gamma_5)s][\bar{Q}\gamma^\mu(1-\gamma_5)Q] \\ &+ \frac{1}{2}[\bar{d}\gamma_\mu(1-\gamma_5)\lambda^j s][\bar{Q}\gamma^\mu(1-\gamma_5)\lambda^j\,Q]\,. \end{aligned} \tag{11.89}$$

The current $\bar{Q}\gamma^\mu(1-\gamma_5)\lambda^j Q$ of the last term in (89) will couple to the QCD current $\bar{Q}\gamma_\mu T^j\,Q$ and leads to a nonvanishing trace in color space.

For each internal quark line Q, one has the following contribution from the loop where the external momenta of s and d are neglected, as in (19),

$$\begin{aligned}\Gamma^\nu_Q(k^2) = \mathrm{i}\left(\frac{-\mathrm{i}g}{2\sqrt{2}}\right)^2\left(\frac{\mathrm{i}}{M_\mathrm{W}^2}\right)(-\mathrm{i}g_\mathrm{s})\,\frac{1}{2}\,[\bar{d}\,\gamma_\mu(1-\gamma_5)\lambda^j\,s\,]\,V^*_{Qd}V_{Qs} \\ \times\,(-)\int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{Tr}\left[\gamma^\mu(1-\gamma_5)\lambda^j\frac{\mathrm{i}}{\not{p}-m_Q}\gamma^\nu\,T^l\frac{\mathrm{i}}{\not{p}-\not{k}-m_\mathrm{Q}}\right].\end{aligned}$$

Here $(-\mathrm{i}g_\mathrm{s})$ and $(-\mathrm{i}g/2\sqrt{2})$ are the strong and weak coupling constants; $\mathrm{i}/M_\mathrm{W}^2$ is the approximation of the W-boson propagator $-\mathrm{i}/(p^2-M_\mathrm{W}^2)$ when we go from Fig. 11.5a to Fig. 11.5b. Its justification will be given later. There is a minus sign due to the anticommutation rule of fermions in the loop, and k is the gluon momentum. Setting $g^2/8M_\mathrm{W}^2 = G_\mathrm{F}/\sqrt{2}$ and using $\mathrm{Tr}\,(\lambda^j\lambda^l) = 2\delta^{jl}$, the above expression becomes

$$\Gamma^\nu_\mathrm{Q}(k^2) = \frac{G_\mathrm{F}}{\sqrt{2}}(-\mathrm{i}g_\mathrm{s})\,[\bar{\mathrm{d}}\,\gamma_\mu(1-\gamma_5)\,T^j\,\mathrm{s}]\,V^*_\mathrm{Qd}V_\mathrm{Qs}\,I^{\mu\nu}_\mathrm{Q}(k^2)\,, \tag{11.90}$$

where

$$I^{\mu\nu}_\mathrm{Q}(k^2) = (-1)\int\frac{\mathrm{d}^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma^\mu(1-\gamma_5)\frac{\mathrm{i}}{\not{p}-m_\mathrm{Q}}\gamma^\nu\frac{\mathrm{i}}{\not{p}-\not{k}-m_\mathrm{Q}}\right]\,. \tag{11.91}$$

Besides the factor $(1-\gamma_5)$ which is irrelevant because of the trace, the above integral is familiar and will be discussed in Chap. 15, in relation with the *vacuum polarization* and the *running coupling*. The divergent part $I^{\mu\nu}_\mathrm{div}$ of

(91), coming from $p \gg M_{\mathrm{W}}$ and $p \gg m_{\mathrm{Q}}$, turns out to be independent of Q. It does not contribute to the transition amplitude because of the unitarity of the CKM matrix, or the GIM cancelation mechanism:

$$\sum_{\mathrm{Q}} V^*_{\mathrm{Qd}} V_{\mathrm{Qs}}\, I^{\mu\nu}_{\mathrm{div}} = I^{\mu\nu}_{\mathrm{div}} \sum_{\mathrm{Q}} V^*_{\mathrm{Qd}} V_{\mathrm{Qs}} = 0 \ .$$

Since the divergences in both diagrams of Fig. 11.5a and Fig. 11.5b vanish by the GIM mechanism, the upper limit of their p-integrals can be taken at any value lower than the W mass, and the substitution $-\mathrm{i}/(p^2 - M_{\mathrm{W}}^2)$ by $\mathrm{i}/M_{\mathrm{W}}^2$ is justified. The W mass plays the role of the momentum cutoff, this in turn explains why the diagram 11.5a gives the same result as the diagram 11.5b.

Only the finite part of the integral (91) remains to be evaluated. The calculation will be done in Chap. 15 and given in (15.6) and (15.30). The dominant finite term can be directly taken from (15.6) which gives

$$I^{\mu\nu}_{\mathrm{Q}}(k^2) = -\mathrm{i}(k^2 g^{\mu\nu} - k^\mu k^\nu)\frac{1}{12\pi^2}\log\frac{m_{\mathrm{Q}}^2}{\mu^2} \ . \tag{11.92}$$

In the course of the computation, a mass scale μ is introduced in (15.6) or (92) for dimensional reason. However μ^2 will disappear in the final result, as can be seen below in (94). Once the loop integral is known, we attach (92) to the external quark current $\overline{q}\gamma_\nu T^j q$ (indicated by C in Fig. 11.5b) via the gluon propagator. Note that the first factor, $k^2 g^{\mu\nu}$, when multiplied by the gluon propagator $-\mathrm{i}/k^2$, yields a local operator, i.e. a k^2-independent finite term. The contribution of the second factor, $k^\mu k^\nu$, vanishes when it operates on the conserved current $\bar{\mathrm{q}}\gamma_\nu T^j \mathrm{q}$.

Finally, summation over internal quarks Q = u, c, t and application of the unitarity of V_{CKM} lead to the penguin operator

$$\begin{aligned} H^{\mathrm{glu}}_{\mathrm{pen}} =& \frac{G_{\mathrm{F}}}{\sqrt{2}}(-\mathrm{i}g_{\mathrm{s}})\ [\bar{d}\,\gamma_\mu(1-\gamma_5)\,T^j\,s] \sum_{\mathrm{Q=u,c,t}} V^*_{\mathrm{Qd}} V_{\mathrm{Qs}} \frac{1}{12\pi^2}\log\frac{m_{\mathrm{Q}}^2}{\mu^2} \\ & \times(-\mathrm{i}k^2 g^{\mu\nu})\left(\frac{-\mathrm{i}}{k^2}\right)\ (-\mathrm{i}g_{\mathrm{s}})\ [\overline{q}\,\gamma_\nu\,T^j\,q] \ , \end{aligned} \tag{11.93}$$

which can also be written as

$$\begin{aligned} H^{\mathrm{glu}}_{\mathrm{pen}} &= \frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{\alpha_{\mathrm{s}}}{12\pi}\left\{V^*_{\mathrm{td}}V_{\mathrm{ts}}\log\frac{m_{\mathrm{t}}^2}{m_{\mathrm{c}}^2} - V^*_{\mathrm{ud}}V_{\mathrm{us}}\log\frac{m_{\mathrm{c}}^2}{m_{\mathrm{u}}^2}\right\}\mathcal{O}_{\mathrm{pen}} \ , \\ \mathcal{O}_{\mathrm{pen}} &= [\bar{d}\,\gamma_\mu(1-\gamma_5)\lambda^j\,s]\ [\overline{q}\,\gamma^\mu\lambda^j\,q] \ . \end{aligned} \tag{11.94}$$

Using $V^*_{\mathrm{cd}}V_{\mathrm{cs}} = -\{V^*_{\mathrm{td}}V_{\mathrm{ts}} + V^*_{\mathrm{ud}}V_{\mathrm{us}}\}$, the term inside the curly brackets {} of (94) is derived from the identity

$$\sum_{\mathrm{Q=u,c,t}} V^*_{\mathrm{Qd}}V_{\mathrm{Qs}}\log\frac{m_{\mathrm{Q}}^2}{\mu^2} = V^*_{\mathrm{td}}V_{\mathrm{ts}}\log\frac{m_{\mathrm{t}}^2}{m_{\mathrm{c}}^2} - V^*_{\mathrm{ud}}V_{\mathrm{us}}\log\frac{m_{\mathrm{c}}^2}{m_{\mathrm{u}}^2} \ .$$

As explicitly shown by (94), this effective s $\to$ d strangeness-changing neutral current is a four-fermion operator in the bilinear form with a total isospin $I = 1/2$, as explained earlier.
The calculation of the imaginary part of $A_0 \equiv \langle \pi(k)\pi(p) \,|\, H^{\rm glu}_{\rm pen} \,|\, \mathrm{K}^0(P)\rangle$ involves the consideration of the matrix element

$$\langle \pi(k)\pi(p) \,|\, [\bar{d}\gamma_\mu(1-\gamma_5)\lambda^j \; s] \; [\bar{q}\gamma^\mu\lambda^j q] \,|\, \mathrm{K}^0(P)\rangle \; . \tag{11.95}$$

From (88), we write $\lambda^j\lambda^j$ in the above equation as a product of the color singlet currents, then applying a Fierz's transformation on the latter, and finally, using the factorization approximation in the calculation of the matrix element [see Sect. 16.4, and (16.95) as an example], we estimate (95) to be

$$\begin{aligned}\langle \pi(k) \,|\, \bar{d}\gamma_\mu\gamma_5 q \,|\, 0\rangle \, \langle \pi(p) \,|\, \bar{q}\gamma^\mu s \,|\, \mathrm{K}^0(P)\rangle & \\ &= f_\pi k_\mu \left[(P+p)^\mu f_+(k^2) + k^\mu f_-(k^2)\right] \\ &= f_\pi \left[(m_{\rm K}^2 - m_\pi^2)\, f_+(m_\pi^2) + m_\pi^2 \, f_-(m_\pi^2)\right] \\ &\approx f_\pi (m_{\rm K}^2 - m_\pi^2) \; ,\end{aligned} \tag{11.96}$$

the last line comes from $f_+(0) \approx 1$. All of these calculations lead to the result

$$\begin{aligned}\mathrm{Im}(A_0) &\approx \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm s}}{12\pi} f_\pi (m_K^2 - m_\pi^2) \mathrm{Im}(V_{\rm td}^* V_{\rm ts}) \log \frac{m_{\rm t}^2}{m_{\rm c}^2} \\ &\approx \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm s}}{12\pi} f_\pi (m_K^2 - m_\pi^2) A^2 \lambda^5 \eta \; \log \frac{m_{\rm t}^2}{m_{\rm c}^2} \, .\end{aligned} \tag{11.97}$$

The term $V_{\rm ud}^* V_{\rm us}$ is real and does not contribute to Im (A_0). With this result for $\mathrm{Im}(A_0)$ and the experimental value for $\mathrm{Re}\, A_0 = 3.3 \times 10^{-4}$ MeV taken from the decay rate of $\mathrm{K_S} \to 2\pi$, one gets ϵ' from (68) using $\omega = 1/22$ and $\mathrm{Im}(A_2) = 0$. For another estimate of A_0, see Problem 16.1.

Finally, from the measured value of $|\epsilon| = 2.258 \times 10^{-3}$, one obtains the ratio $|\epsilon'/\epsilon| \approx 10^{-3} A^2 \eta$. Thus the standard model predicts a small but definitely nonzero ratio $|\epsilon'/\epsilon| \approx 10^{-4}$ with a large uncertainty by a factor of 3, the uncertainty essentially comes from the difficult evaluation of the matrix element in (95) because of its nonperturbative character. Nevertheless, this prediction provides an important test of the CP violation in the KM fashion. A ratio that is either vanishingly small or greater than about 10^{-3} would indicate that this mechanism is inadequate and that an explanation beyond the standard model is called for. Experimental measurements of ϵ' are being planned at CERN and FNAL. The ultimate accuracy of these experiments would be better by an order of magnitude than (69), and may resolve this important issue.

Problems

11.1 CP-even and -odd eigenvalues of pions in K^0 decay. We consider the neutral K meson decays into pions. Show that the two-pion system $\pi^0\pi^0$ and $\pi^+\pi^-$ has even eigenvalue $\mathcal{CP}\,|\pi\pi\rangle = +\,|\pi\pi\rangle$. Also show that the three-pion system $\pi^0\pi^0\pi^0$ has odd eigenvalue $\mathcal{CP}\,\left|\pi^0\pi^0\pi^0\right\rangle = -\,\left|\pi^0\pi^0\pi^0\right\rangle$. How about the CP eigenvalue of $\pi^+\pi^-\pi^0$?

11.2 $\Delta I = 1/2$ rule in the decays of strange particles. Show that $\Gamma(\mathrm{K}^0 \to \pi^+ + \pi^-) = 2\Gamma(\mathrm{K}^0 \to \pi^0 + \pi^0)$, if $\Delta I = 1/2$ strictly holds. The deviation is used to measure the ratio $\omega = |A_2/A_0| = 1/22$ of the amplitudes A_0 and A_2 mentioned in the text. With the $\Delta I = 1/2$ rule, show that the amplitudes $a_+ \equiv \mathcal{A}(\Sigma^+ \to \mathrm{n} + \pi^+)$, $a_- \equiv \mathcal{A}(\Sigma^- \to \mathrm{n} + \pi^-)$, and $a_0 \equiv \mathcal{A}(\Sigma^+ \to \mathrm{p} + \pi^0)$ satisfy $a_+ + \sqrt{2}\,a_0 = a_-$. This relation, represented by a rectangular triangle, can be translated into $\Gamma(\Sigma^+ \to \mathrm{n}+\pi^+) = \Gamma(\Sigma^- \to \mathrm{n} + \pi^-) = \Gamma(\Sigma^+ \to \mathrm{p} + \pi^0)$. Compare this prediction with the data.

11.3 Long and short neutral D and B mesons. Explain why for the flavored neutral meson systems : $\mathrm{D}^0 = (\overline{\mathrm{u}}\,\mathrm{c})$, $\mathrm{B}^0_\mathrm{d}(\overline{\mathrm{b}}\mathrm{d})$, and $\mathrm{B}^0_\mathrm{s}(\overline{\mathrm{b}}\mathrm{s})$, the CP-even and -odd eigenstates cannot appear as the long and short components to be easily identified, contrary to the neutral K system.

11.4 Mass difference Δm_B. For the two eigenstates coming from the B^0_d–$\overline{\mathrm{B}}^0_\mathrm{d}$ mixing, while one cannot make a distinction between their lifetimes, their mass difference Δm_B can however be measured (Chap. 16). It turns out that $\Delta m_\mathrm{B} = (3 \pm 0.12) \times 10^{-4}$ eV $\approx 10^2 \times \Delta m_\mathrm{K}$. With such value of Δm_B, show that one can predict a lower bound of the top quark mass, before its discovery in 1994. Explain why the mass difference $\Delta m^\mathrm{s}_\mathrm{B}$ of the two B^0_s eigenstates is again much larger than Δm_B. Estimate $\Delta m^\mathrm{s}_\mathrm{B}$.

11.5 The relation (11.88). Derive this useful relation.

Suggestions for Further Reading

$\mathrm{K_L}$, $\mathrm{K_S}$*, strangeness oscillations, regeneration:*

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